Integrated Math 2 Date: Solutions

8B – Compound Events

❖ Vocabulary, Formulas, Theories:

- **Compound Event**: an event that is made up of two or more events.
- **Intersection:** a compound event when P(A) and P(B) will happen. It's written like this: $P(A \text{ and } B)$. If A and B are independent, the following formula is used:

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

- **Mutually Exclusive Events (disjoint)**: events that cannot happen at the same time. For example, you cannot roll a 2 and a 5 on a standard number cube at the same time. These events are mutually exclusive. The probability of both happening is zero. If events A and B are mutually exclusive, then $P(A \text{ and } B) = 0$.
- **Union**: a compound event when P(A) or P(B) will happen. It's written like this: $P(A \text{ or } B)$. If the events are mutually exclusive, the following formula can be used:

$$P(A \text{ or } B) = P(A) + P(B)$$

- **Overlapping Events (non-disjoint)**: events that have outcomes in common. For example, when rolling a standard number cube, the events of an even number and a multiple of 3 overlap because the number 6 meets both conditions. If events A and B are overlapping, they follow this formula:

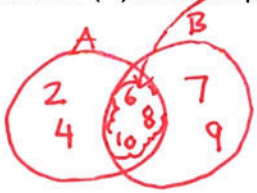
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

🎞 Video #1 - "Probability - Intersection and Union - Example" - Don't Memorise (4:12)

EX1) Jack has a bag of numbers 1 through 10 and he is going to select a number from the bag. The probability of him pulling an even number will be P(A) and the probability of him pulling a number greater than 5 will be P(B).

a) Find P(A and B)
b) Find P(A or B)

A: even # → {2, 4, 6, 8, 10}
B: # > 5 → {6, 7, 8, 9, 10}



$P(A \text{ and } B) = \frac{3}{10}$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$= \frac{5}{10} + \frac{5}{10} - \frac{3}{10}$

$= \frac{5+5-3}{10} = \frac{7}{10}$

🎞 Video #2 - "Multiplication Rules Probability Independent Events" - Daniel Schaben (8:58)

EX2) Frankie flips a coin and rolls a six-sided number cube. What is the probability that he flips heads and rolls a 5?

A: rolls 5
B: flips heads

$P(A \text{ and } B) = P(A) \cdot P(B)$

$= \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}$

Sample space of cube = {1, 2, 3, 4, 5, 6}
Sample space of coin = {H, T}

EX3) Given a standard deck of 52 cards, Maria has to randomly pick one card, put it back, and pick a second card. What are the chances of picking a diamond her first pick and a king on her second pick? What about if she didn't replace the card after her first pick?

A: picks ◇
B: picks king

$P(\diamond \text{ and } K) = P(A \text{ and } B) = P(A) \cdot P(B)$

$= \frac{13}{52} \cdot \frac{4}{52}$

$= \frac{1}{52}$

WITHOUT REPLACEMENT

$\frac{13}{52} \cdot \frac{4}{51} = \frac{1}{51}$

EX4) John needs to pick two marbles out of a bag of 3 red marbles and 2 black marbles. If he picks a marble, puts it back and picks another marble, find the following probabilities:

Total = 3+2 = 5 marbles

a. two red marbles
b. two black marbles
c. red and a black

a. P(2 red marbles)
= P(pick red AND pick red again)
= P(red) · P(red)
= $\frac{3}{5} \cdot \frac{3}{5}$
= $\boxed{9/25}$

b. P(2 black)
= P(black) · P(black)
= $\frac{2}{5} \cdot \frac{2}{5}$
= $\boxed{4/25}$

c. P(red and a black)
= P(red) · P(black)
= $\frac{3}{5} \cdot \frac{2}{5}$
= $\boxed{\frac{6}{25}}$

Video #3 - "Addition Rules for Probability" - Daniel Schaben (14:49)

EX5) Given a standard deck of 52 cards, find the following probabilities using the given events.

A = picking an ace B = picking a king C = picking a club
E = picking a red F = picking a black

Notice no intersection

a. P(A or B)
b. P(A or C)
c. P(A or E)

* See sample space diagram

a. P(A or B)
= P(A) + P(B)
= $\frac{4}{52} + \frac{4}{52}$
= $8/52 = \boxed{\frac{2}{13}}$

b. P(A or C) OVERLAPPING EVENTS
= P(A) + P(C) − P(A and C)
= $\frac{4}{52} + \frac{13}{52} - \frac{1}{52}$
= $\frac{16}{52} = \boxed{\frac{4}{13}}$

c. P(A or E)
= P(A) + P(E) − P(A and E)
= $\frac{4}{52} + \frac{26}{52} - \frac{2}{52}$
= $\frac{28}{52} = \boxed{\frac{7}{13}}$

EX6) Given that two six sided dice were rolled, find the following probabilities using the given events.

A = sum of 7 B = both are even C = rolling at least one 3

a. P(A or B)
b. P(A or C)

a. P(A or B)
= P(A) + P(B)
= $\frac{6}{36} + \frac{9}{36}$
= $\frac{15}{36} = \boxed{\frac{5}{12}}$

b. P(A or C) ← OVERLAPPING EVENTS
= P(A) + P(C) − P(A and C)
= $\frac{6}{36} + \frac{11}{36} - \frac{2}{36}$
= $\frac{15}{36} = \boxed{\frac{5}{12}}$

❖ Extra Resources:

https://www.youtube.com/watch?v=v1CB9eA2XvE

https://www.youtube.com/watch?v=DOooyE6liLY

Example 6

a.

		Second throw					
		1	2	3	4	5	6
First throw	1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

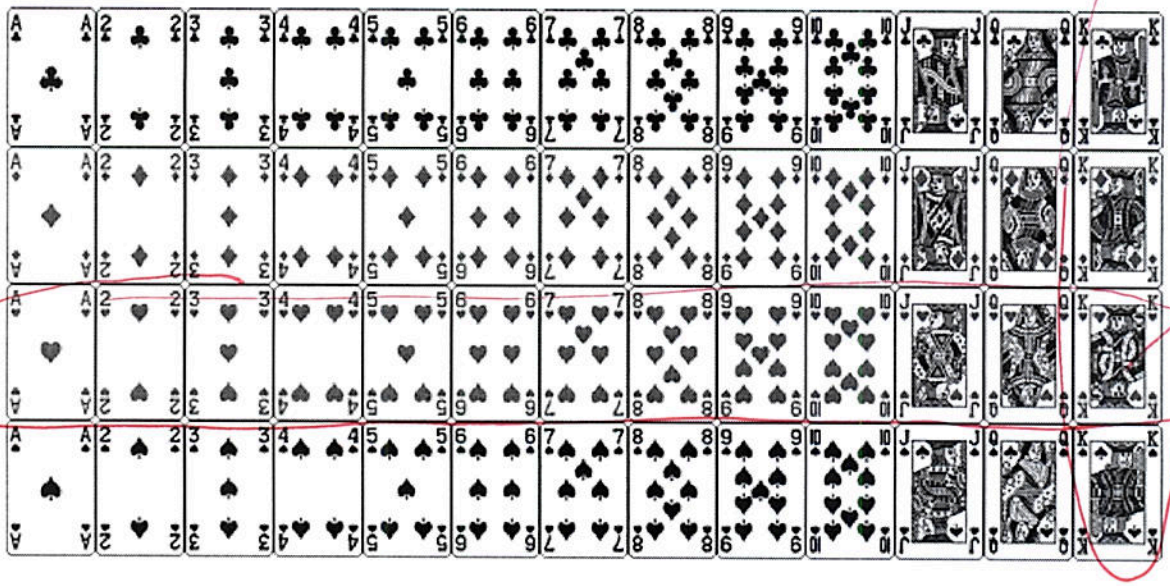
Sum of 7

b.

		Second throw					
		1	2	3	4	5	6
First throw	1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

		Second throw					
		1	2	3	4	5	6
First throw	1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
	2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
	3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
	4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
	5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
	6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

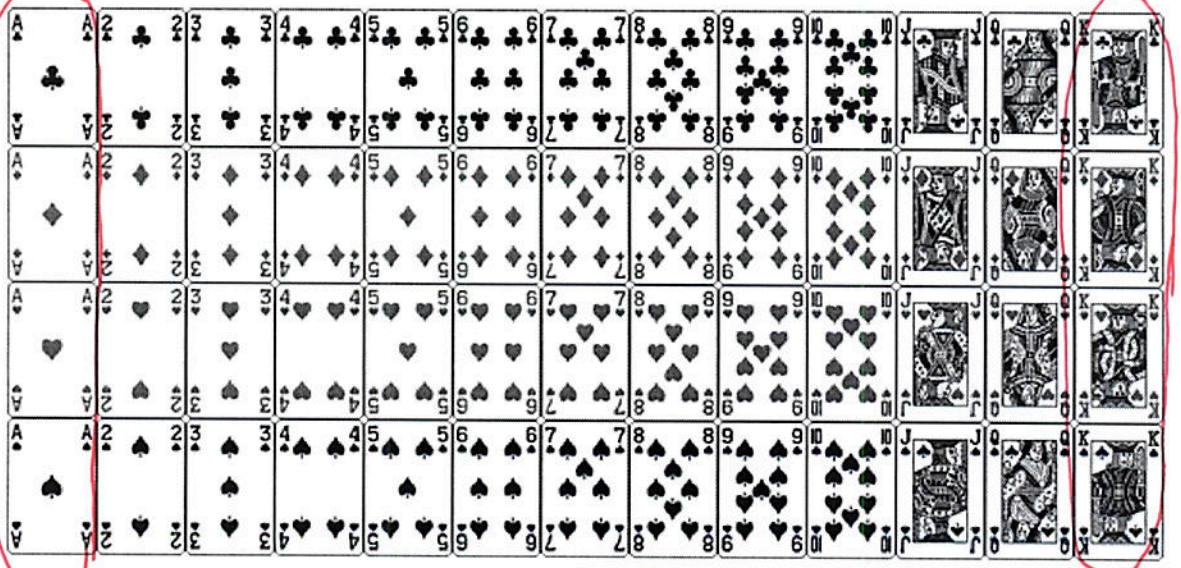
Checkpoint 8B #6



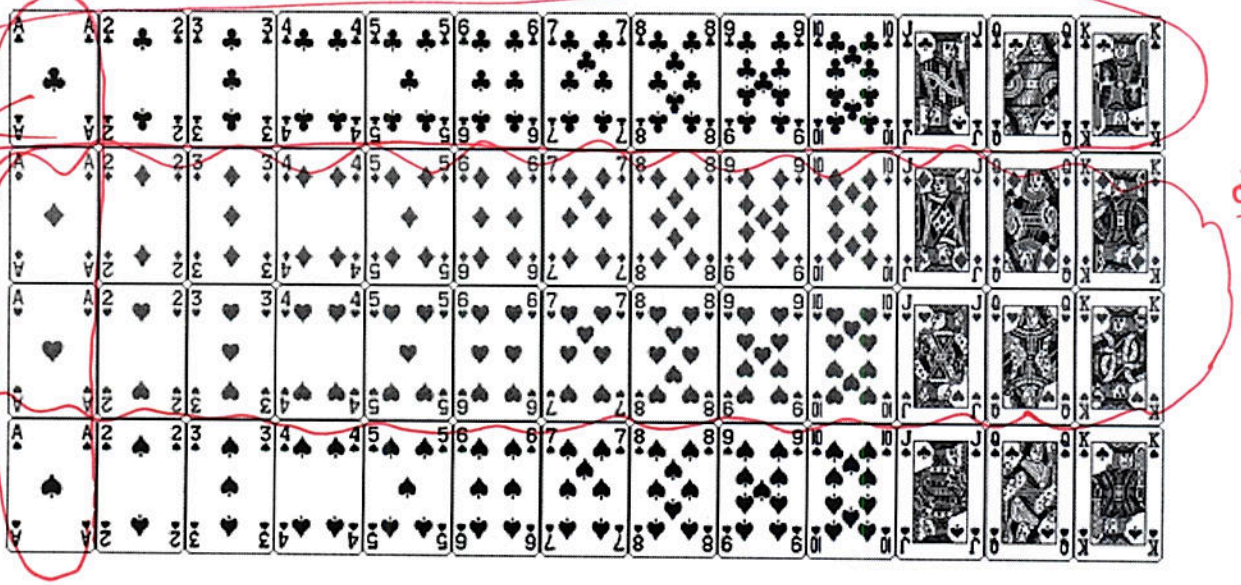
1 intersection

Example 5

a)



b)



intersection

c)