Integrated Math 2 Date: Solutions

8D – Conditional Probability

❖ Vocabulary, Formulas, Theories:

- **Conditional Probability**: a probability that is calculated based on the assumption that some event has already occurred. It follows a certain notation. For example, the probably of Event B happening given that Event A has occurred is written as P(B | A).

For any two events A and B, the probability of B occurring, given that event A has occurred, is $P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$, where $P(A) \neq 0$.

- **Conditional Probability to Test for Independence**: conditional probability can be used to determine if two events are independent or not:

Independent if $P(A \mid B) = P(A)$ or if $P(B \mid A) = P(B)$

If $P(B \mid A) = P(B)$, then the statement $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$ simplifies to $P(A \text{ and } B) = P(A) \cdot P(B)$, which is true only if A and B are independent events. This leads to another way to test events for independence:

Video - "Conditional Probability" - Mathispower4u (7:51)

EX1) You roll one 6 sided die, what is the probability of a 3 given you know the number is odd?

A: roll odd #

B: roll 3

Method I:

$P(B|A) = \boxed{\frac{1}{3}}$

Method II:

$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{6}}{\frac{3}{6}} = \frac{1}{\cancel{6}} \cdot \frac{\cancel{6}}{3} = \boxed{\frac{1}{3}}$

Odd # → {1, 3, 5}

EX2) At P-Town High School, the probability that students takes Computer Programming and Spanish is 0.15. The probability that a student takes Computer Programming is 0.4. What is the probability that a student takes Spanish given that the student is taking Computer Programming?

A: takes Spanish

B: take Computer Prog.

$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{0.15}{0.40} = \frac{15}{40} = \boxed{\frac{3}{8}}$ or $\boxed{0.375}$

EX3) A survey was given to adults parents with children. The results are in the two way table. What is the probability that a person thinks college is too expensive given they have a child in college?

P(too expensive | child in college)

$= \boxed{\frac{55}{59}}$

	Too Expensive	Affordable	Too Cheap	Total
Child in College	55%	4%	0%	59%
Child not in College	30%	8%	3%	41%
Total	85%	12%	3%	100%

EX4) Use a standard deck of 52 cards to find the probabilities:

a) Two cards are drawn without replacement in succession. What is the probability that the second card drawn is an ace, given that the first card drawn was an ace?

b) Two cards are drawn without replacement. What is the probability the second card is a red face card given the first card is a red face card?

a) A: draw ace
B: draw another ace

$P(B|A) = \frac{3}{51}$

b) $P(2^{nd}$ card red face card $|$ 1^{st} card is red face card$)$
$= \frac{5}{51}$

FACE CARDS: K, Q, J → 6 red

Video - "AP Stats Conditional Probability & Independence" - mathcox (8:11)

EX5) Suppose 32% of American adults have blonde hair, 48% have brown eyes, and 21% have both. Use conditional probability to determine if having blonde hair and having brown eyes are independent events.

A: Blonde hair
B: Brown eyes
A and B: Blonde hair and Brown eyes

Check either: $P(A|B) = P(A)$ or $P(B|A) = P(B)$

I'll check the 1^{st} one: $P(A|B) = P(A) \Rightarrow 0.4375 \neq 0.32$

$P(A) = 0.32$

$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{0.21}{0.48} = 0.4375$

∴ A, B are not independent.

EX6) Given that 75% of American Adults have children, 60% are married, and 45% are both married and have children, use conditional probability to determine if being married and having children are independent events.

A: have children
B: are married

$P(A|B) = P(A)$?
✓ $0.75 = 0.75$

$\begin{cases} P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{0.45}{0.60} = \frac{45}{60} = 0.75 \\ P(A) = 0.75 \end{cases}$

∴ A, B are independent events

Video - "How to Find Conditional Probabilities and Determine if Events are Independent" - MathBootcamps (6:52)

EX7) A group of full time and part time employees were asked if they were interested in additional training or not. Use the results to determine if "an employee being full time" and "an employee interested in additional training" are independent.

	Full Time	Part Time	Total
Interested in Additional Training	66	83	149
Not Interested in Additional Training	76	62	138
Total	142	145	287

A: employee being full time
B: employee interested in add. training

CHECK: $P(A|B) = P(A)$

$\frac{66}{149} \stackrel{?}{=} \frac{142}{287}$

$0.44 \neq 0.49$

$P(A) = \frac{142}{287}$

$P(A|B) = \frac{66}{149}$

∴ A, B are not independent events (they are dependent)

❖ Extra Resources:
https://www.youtube.com/watch?v=-iYV42YKN44