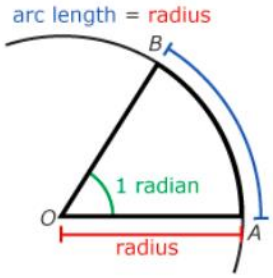
Integrated Math 2 Date: _____

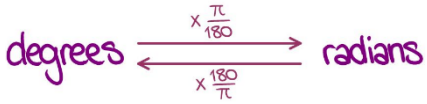
9C – Arc Length and Area

❖ Vocabulary, Formulas, Theories:

- **Radian**: a unit for measuring angles often written in terms of π. The number of radians in an angle equals the number of radii it takes to measure a circular arc described by that angle. For example, 1 rad is when the length of the intercepted arc is equal to the radius of the circle. Click here for a short animated demonstration of a radian.



- **Converting Degrees and Radians**: the unit of angles can be switched between degrees and radians using the ratios $\frac{\pi}{180^\circ}$ and $\frac{180^\circ}{\pi}$.



- **Measure of an Arc**: the measure of an arc on a circle. It's associated with but different than a length of an arc.

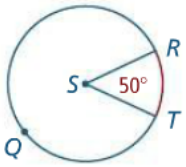
Arc Measure

The measure of a minor arc is equal to the measure of its corresponding central angle.

The measure of a major arc is the measure of the related minor arc subtracted from 360.

The measure of a semicircle is 180.

Example



$m\overset{\frown}{RT} = m\angle RST = 50$

$m\overset{\frown}{TQR} = 360 - m\overset{\frown}{RT}$

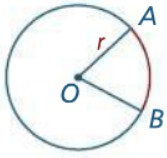
$= 310$

- **Length of an Arc**: the length of an arc on a circle. It's associated with but different than a measure of an arc.

The length of an arc of a circle is the product of the ratio $\frac{\text{measure of the arc}}{360}$ and the circumference of the circle.

$$\text{length of } \overset{\frown}{AB} = \frac{m\overset{\frown}{AB}}{360} \cdot 2\pi r$$

$$= \frac{m\overset{\frown}{AB}}{360} \cdot \pi d$$



Up until this point, angles have been measured in degrees. Now there will be a new unit called a radian. A radian is the measure of an angle when the intercepted arc is equal to the radius of the circle. For example, if the radius of a circle is 5 inches, the angle of 1 radian will be when the arc length is also 5 inches. Click here for a short animated demonstration of a radian.

➢ Extra: Click here for a goofy video about degrees and radians of a unit circle.

Video - "Examples: Converting Angles in Degree Measure to Radian Measure" - Mathispower4u (3:02)

EX1) Convert the given angles from “degrees to radians.”

a) 135° b) 15°

Video - "Examples: Convert Angles in Radian Measure to Degree Measure" - Mathispower4U (2:35)

EX2) Convert the given angles from “radians to degrees.”

a) $\frac{5\pi}{6}$ b) $\frac{2\pi}{9}$

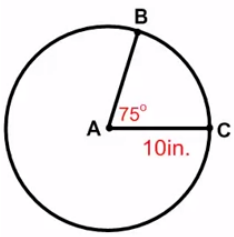
The measure of an arc was previously determined by the central angle that created the particular arc being measure. For example, a central angle of 25 degrees would create an intercepted arc that would have a measure of 25 degree. The measure of an arc isn’t the only part of an arc that can be calculated. The length can also be determined. This measurement is slightly different.

EX3) Derive (create) the formula for the length of an arc.

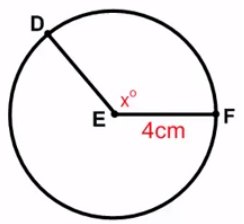
Now that there has been a formula created, use it to calculate the length of an arc given some information about a circle.

Video - "Finding Arc Length of a Circle" - TheDouceHouse (9:30)

EX4) Determine the length of BC.



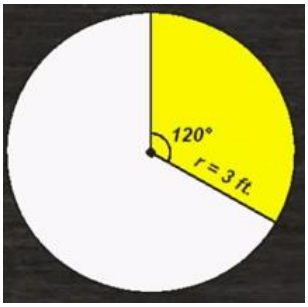
EX5) Determine the central angle x (to the nearest degree) if DF has an arc length of 9 cm.



Circles are often divided into multiple parts depending on the application. These parts have their own areas which can be measured. The next video does a good job of introducing that idea.

Video - "How To Find The Area Of A Circle's Sector" - mrmaisonet (3:57)

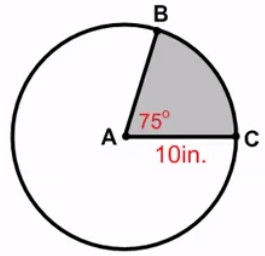
EX6) Determine the area of the yellow region, which was created by the 120° central angle.



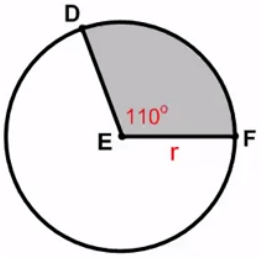
This next video goes through more examples involving the area of a section. It uses a proportion to solve, so it's a little different than the process show in the above video.

Video - "Finding Sector Area of a Circle" - TheDouceHouse (8:31)

EX7) Determine the area of the shaded region.



EX8) Determine the radius of the circle if the area of the shaded region is 50π.



❖ Extra Resources:

http://www.regentsprep.org/regents/math/algtrig/atm1/arclengthlesson.htm