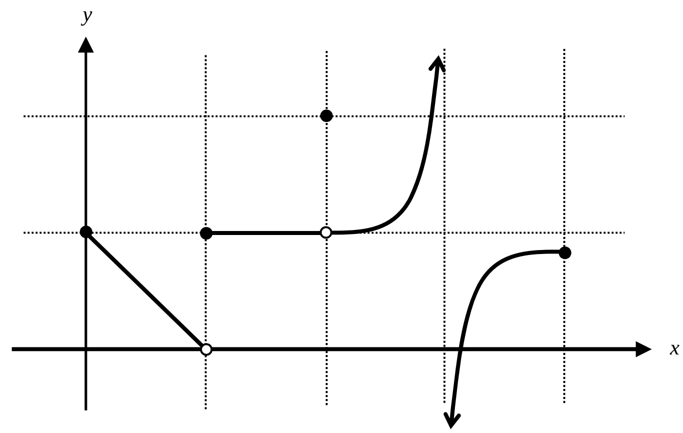
Pre Calculus
Worksheet 1.2 Day 1

1. The relation described by the set of points $\{(-2,5),(0,5),(3,8),(3,9)\}$ is NOT a function. Explain why.

For questions 2 - 4, use the graph at the right.



2. Explain why the graph represents a function.

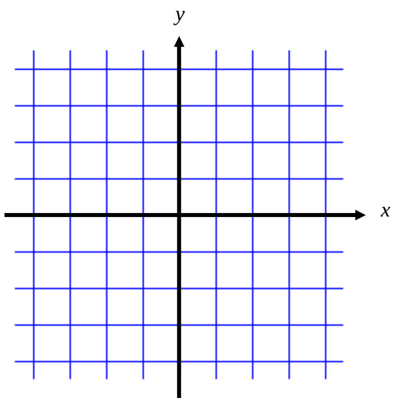
3. Where is the function above discontinuous.
   Describe each type of discontinuity.

4. Using interval notation, describe the domain and range of the function above?

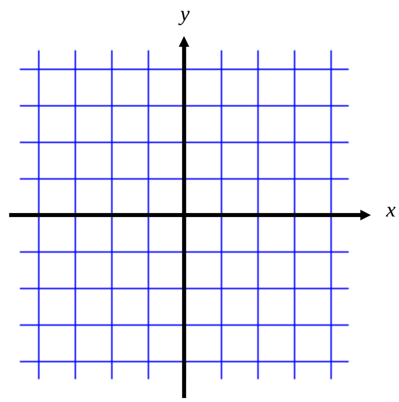
5. What are the 3 domain issues you must remember in this course?

6. Graph each of the following functions. What do you notice? What happens when $x = 2$ on the graph of b?

a) $f(x) = x + 2$



b) $g(x) = \frac{x^2 - 4}{x - 2}$



7. What is the domain and range of the two functions above?

8. For each of the following functions, describe the domain in interval notation and then, for any values NOT in the domain, identify the type of discontinuity (if they exist). You need to find the domain without using your calculator. You MAY use your calculator to determine the type of discontinuity, but we will find the discontinuity algebraically later this semester.

a) $f(x) = x^3 - 2x^2 + 1$

Domain:

Discontinuity:

b) $g(x) = \log_3(x-4)$

Domain:

Discontinuity:

c) $h(x) = \dfrac{\sqrt{x+4}}{x-3}$

Domain:

Discontinuity:

d) $k(x) = \dfrac{x^2 - 2x}{x}$

Domain:

Discontinuity:

e) $h(x) = \dfrac{8x^2 - 14x - 15}{6x^2 - 13x - 5}$

Domain:

Discontinuity:

f) $p(x) = \dfrac{3x^2 + 5x + 2}{(x+1)\sqrt{2x+9}}$

Domain:

Discontinuity:

g) $j(x) = \dfrac{1}{x} + \dfrac{5}{x-2}$

Domain:

Discontinuity:

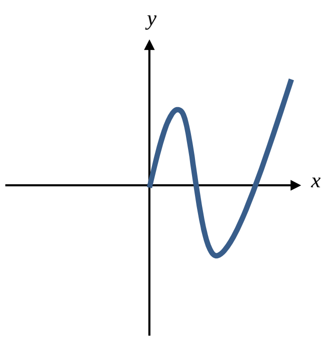
h) $q(x) = \dfrac{\sqrt{4-x^2}}{x-3}$

Domain:

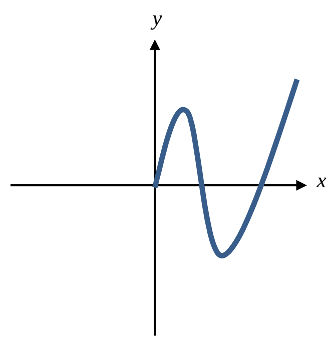
Discontinuity:

9. The given function is only drawn for $x \geq 0$. Complete the function for $x < 0$ with the following conditions:

a) the function is ODD



b) the function is EVEN



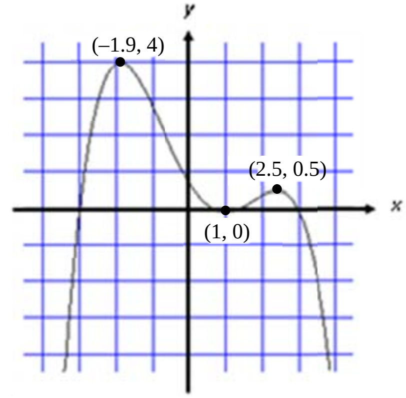
10. Suppose you know the point (–2, –10) is on the graph of a function.

a) If the function is ODD, what other point is on the function? _____________

b) If the function is EVEN, what other point is on the function? ____________

11. Use the graph at the right to answer the following questions:

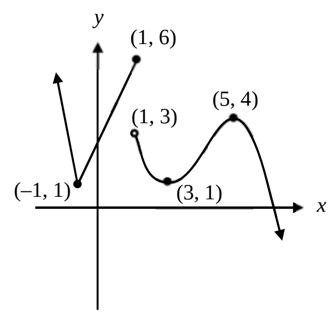
a) Identify all extrema.



b) Identify the intervals on which the function is increasing and decreasing.

12. Use the graph at the right to answer the following questions:

a) Identify all extrema.



b) Identify the intervals on which the function is increasing and decreasing.

13. Use your graphing calculator to graph the function $g(x) = -x^3 + 2x - 3$.

a) Identify all extrema.

b) Identify the intervals on which the function is increasing and decreasing.

14. Determine whether the following functions are bounded above, bounded below, bounded, or not bounded.

a) $y = 32$ b) $y = 2^x$ c) $y = 2 - x^2$ d) $y = \sqrt{1 - x^2}$ e) $y = x^2\sqrt{x + 4}$

Pre Calculus
Worksheet 1.2 Day 2

1. Let $f(x) = -2x^3 + 7x$ and $g(x) = |2x| - 5$. Answer the following.

   a) $-f(x)$ b) $f(-x)$ c) $g(-a)$ d) $-g(a)$

2. How do you algebraically prove a function is ODD or EVEN ?

3. Prove whether each function is even, odd or neither. SHOW ALL STEPS!!

   a) $f(x) = \sqrt{x^2 + 2}$ b) $g(x) = 2x^3 - 3x$ c) $h(x) = -x^2 + 0.03x + 5$

   d) $k(x) = x^3 + 4.2x^2 - 7$ e) $g(x) = \dfrac{3x^4}{1 + x^2}$ f) $j(x) = \dfrac{5}{|x|}$

4. Write the end behavior of the function using limit notation. Graphing calculator allowed.

   a) $f(x) = -x^3 - 2$ b) $f(x) = xe^{-x}$ c) $f(x) = |3 - x|$

   d) $f(x) = \dfrac{-x^3}{x - 1}$ e) $f(x) = \dfrac{4x - 1}{x + 3}$ f) $f(x) = \dfrac{4x}{x^2 + 1}$

5. The last three functions above are called Rational Functions.

   a) Explain why these functions are named so.

   b) What do you notice about the end behavior of #4d as compared to the other rational functions?

6. For a rational function, how can you tell whether a discontinuity is a hole or a vertical asymptote?

7. For each function below, find the following WITHOUT A CALCULATOR:

i) Domain
ii) Vertical Asymptotes or Holes

a) $f(x)=\dfrac{2x-1}{3x+5}$

b) $g(x)=\dfrac{(3x-5)(x-8)}{x^2-4}$

c) $h(x)=\dfrac{2x-9}{x^2-x-6}$

d) $k(x)=\dfrac{2x+4}{x^2-3x-10}$

e) $p(x)=\dfrac{x+2}{3-x}$

f) $q(x)=\dfrac{4}{x^2+9}$

9. Sketch a freehand graph of a function with domain $(-\infty, 0) \cup (0, \infty)$ that satisfies ALL of the listed conditions.

a) f has a non-removable discontinuity at $x = 0$ and vertical asymptote $x = 0$
b) f has a relative maximum of -3 at $x = 5$ and an absolute minimum of -5 at $x = -2$
c) f has a removable discontinuity at $x = 7$
d) $\lim\limits_{x\to-\infty} f(x)=\infty \quad \lim\limits_{x\to\infty} f(x)=\infty$

10. Sketch a freehand graph of a function with domain $(-\infty, \infty)$ that satisfies ALL of the listed conditions.

a) f is continuous for all x
b) $f(-x) = f(x)$
c) f is increasing on $[0, 2)$ and decreasing on $[2, \infty)$
d) $f(2) = 3$

Pre Calculus
Worksheet 1.3

1. Sketch the 12 basic functions from memory.

$y = x$ $y = x^2$ $y = x^3$ $y = \sqrt{x}$ $y = \frac{1}{x}$

$y = b^x$ $y = \log_b x$ $y = \sin x$ $y = \cos x$ $y = |x|$

$y = \lfloor x \rfloor$ $y = \frac{1}{1+e^{-x}}$

Use the equation(s) above (not the names) to answer questions 2 – 15.

2. Seven of the twelve basic functions have the property that $f(0) = 0$. Which five do not?

3. Identify the four basic functions that are odd.

4. How many of the twelve basic functions are even? List them.

5. Identify the six basic functions that are increasing on their entire domains.

6. Identify the three basic functions that are decreasing on the interval $(-\infty, 0)$.

7. Only 3 of the twelve basic functions are bounded. Which three?

8. Which of the twelve basic functions are not continuous? Identify the types of discontinuity in each function.

9. Identify the three basic functions with no zeros.

10. How many of the twelve basic functions have a range of all real numbers? List them.

11. Identify the four functions that do NOT have end behavior $\lim_{x \to \infty} f(x) = \infty$.

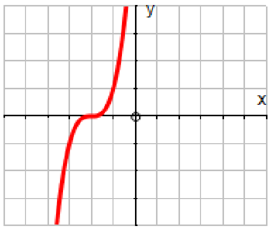
12. How many of the twelve basic functions have end behavior $\lim_{x \to -\infty} f(x) = -\infty$? List them.

13. How many of the twelve basic functions look the same when flipped about the *y*-axis? List them.

14. How many of the twelve basic functions look the same upside down as right-side up? List them.

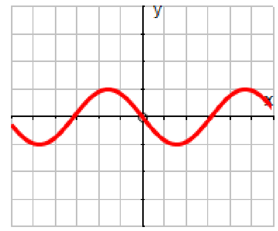
15. How many of the twelve basic functions are bounded below? List them.

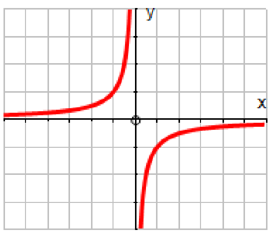
16. Each of the ten graphs below is a slight variation (a transformation) of one of the parent functions. Match the graph to the correct equation.



$y = -\sin(x)$

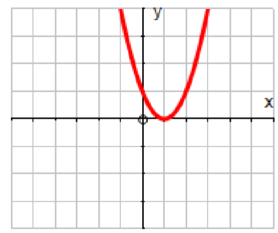
$y = \cos(x) + 1$

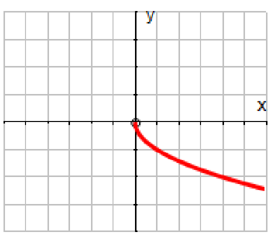




$y = e^x - 2$

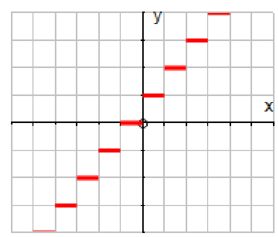
$y = (x+2)^3$

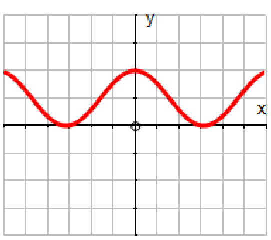




$y = (x-1)^2$

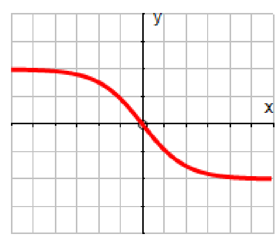
$y = |x| - 2$

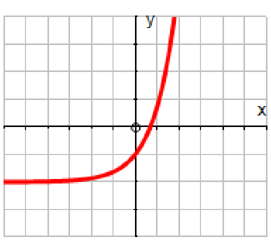




$y = -\frac{1}{x}$

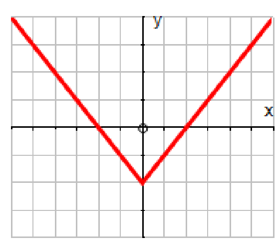
$y = -\sqrt{x}$





$y = \lfloor x+1 \rfloor$

$y = 2 - \frac{4}{1+e^{-x}}$



17. Graph each of the functions below on your calculator, then answer questions *i* and *ii*:

i) How does the graph relate to a graph of one of the twelve basic functions.

ii) Identify any extrema, if they exist.

a) $r(x)=\sqrt{x-10}$

b) $q(x)=e^{x}+2$

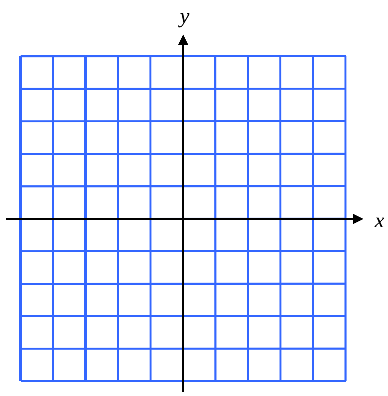
c) $h(x)=|x|-10$

18. The graph of $f(x)=\sqrt{x^2}$ is one of the twelve basic functions. Guess which one, then graph $f(x)$ on your calculator. Were you right? If not, which of the basic twelve functions is $f(x)$?

Piecewise Functions

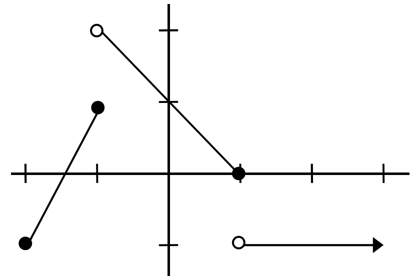
Piecewise Functions are simply functions that have been broken into 2 or more "pieces" where each piece is a portion of the graph with a limited domain. The limitations on the domain allow for the overall equation to pass the vertical line test, and thus be called a function.

19. The function $y=|x|$ can be written as a piecewise function. Draw a graph of $y=|x|$, and then fill in the blanks below with the appropriate domain for each piece to complete the piecewise function representation of $y=|x|$.



$$y=\begin{cases} x & \text{if } \_\_\_\_\_\_\_\_ \\ -x & \text{if } \_\_\_\_\_\_\_\_ \end{cases}$$

20. Write a piecewise function for the graph at the right.

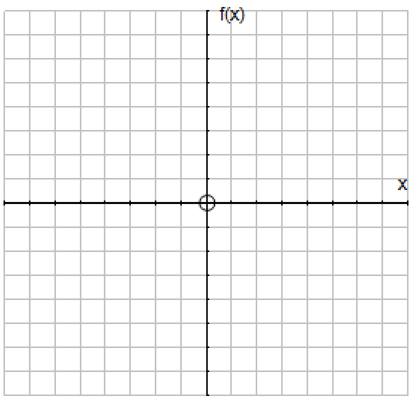


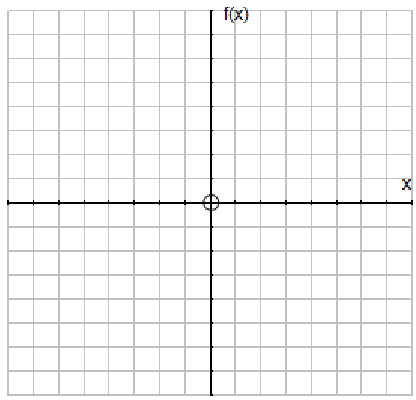
21. Sketch the graph of each piecewise-define function without a calculator.
Be sure to ask yourself … "Self, do my graphs pass the vertical line test?".

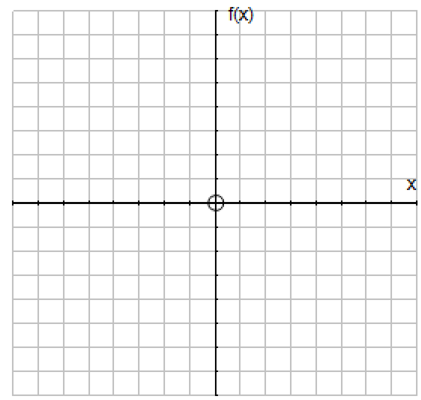
a) $f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 0 \\ \sqrt{x} & \text{if } x \geq 0 \end{cases}$

b) $f(x) = \begin{cases} -3 - x & \text{if } x \leq 0 \\ 1 & \text{if } 0 < x < 1 \\ x^2 & \text{if } x \geq 1 \end{cases}$

c) $f(x) = \begin{cases} x^3 & \text{if } x < -1 \\ |x| & \text{if } -1 \leq x < 1 \\ \lfloor x \rfloor & \text{if } x \geq 1 \end{cases}$







22. At a recent softball game Mr. Leckie hit a double. He ran from home to second base (a distance of 120 feet) at a speed of 20 feet/sec. The next batter was up to bat for 1 minute but popped up to the pitcher, so Mr. Leckie didn't go anywhere. The 3rd batter hit the first pitch over the fence for a homerun, so Mr. Leckie was able to jog home (a distance of 120 feet) at a speed of 10 feet/sec. Write a piecewise function for the distance in feet Mr. Leckie traveled as a function of time in seconds.

23. An earthquake that occurred at 9:17 AM cracked a water tower in a small town. Water began leaking out of the tower at a rate of 12 cm^3/min for the first 30 minutes, then the rate increased to 25 cm^3/min for the next 40 minutes before they leak was fixed. Write a piecewise function for the amount of water *W* that leaked out of the tower as a function of time *t*. (What time should you let $t = 0$ represent? ____________)

24. Two functions are said to be inverses of each other (more on this in section 1.4) if the graph of one can be obtained from the other by reflecting it across the line $y = x$.

a) Two of the twelve basic functions in this section are inverses of each other. Which are they?

b) Two of the twelve basic functions in this section are their own inverses. Which are they?

c) If you restrict the domain of one of the twelve basic functions to $[0, \infty)$, it becomes the inverse of another one. Which are they?

Pre Calculus
Worksheet 1.5

For questions 1 – 8, find the following for each function:

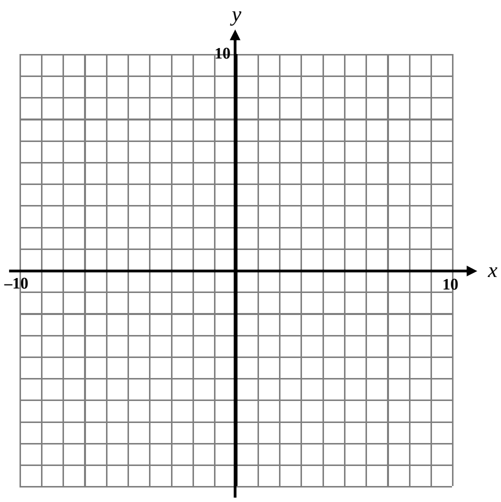
- **a) Identify the parent function and describe the transformation**
- **b) Accurately graph the function without using your calculator**
- **c) Identify the domain and range**

1. $y = 3\log_2(x+5)$

a) ______________________

____

b)



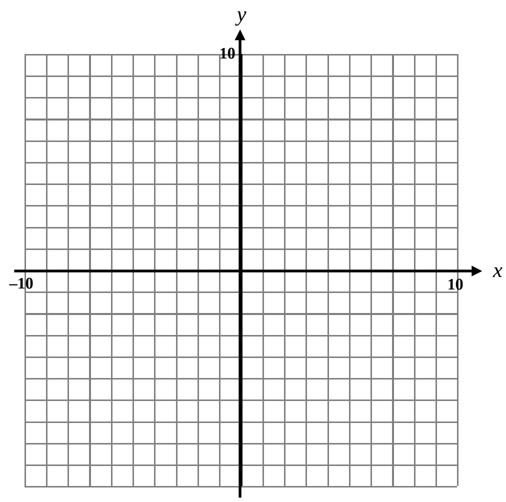
c) ______________________________________

2. $y = (x-2)^3 + 1$

a) ______________________

____

b)



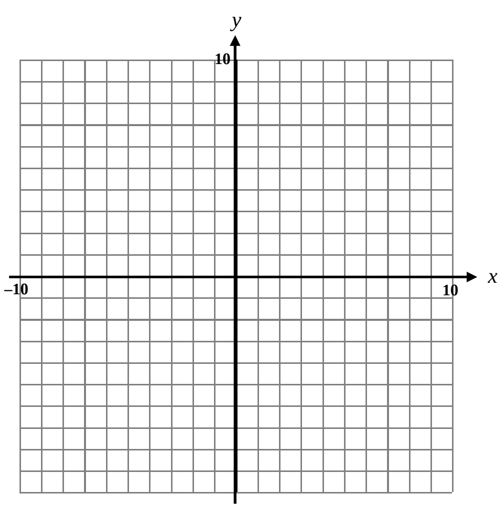
c) ______________________________________

3. $y = 2^{x/4} - 3$

a) ______________________

____

b)



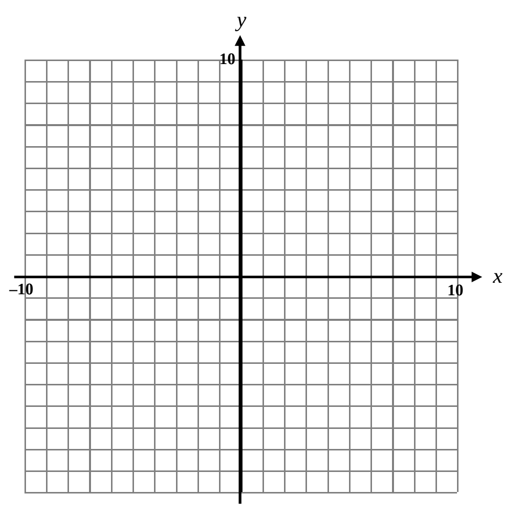
c) ______________________________________

4. $y = 3\lfloor x - 1 \rfloor$

a) ______________________

____

b)



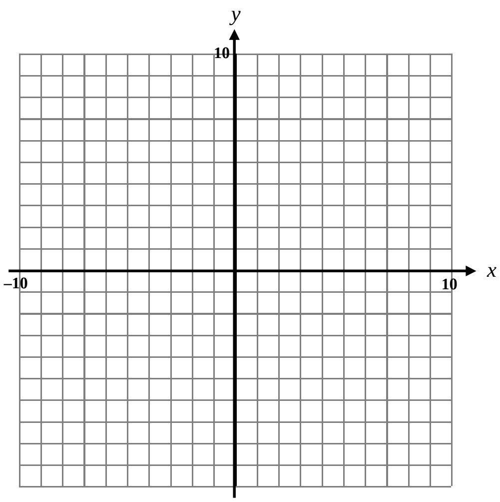
c) ______________________________________

5. $y = |2x + 5| - 3$

a) ________________________

____

b)



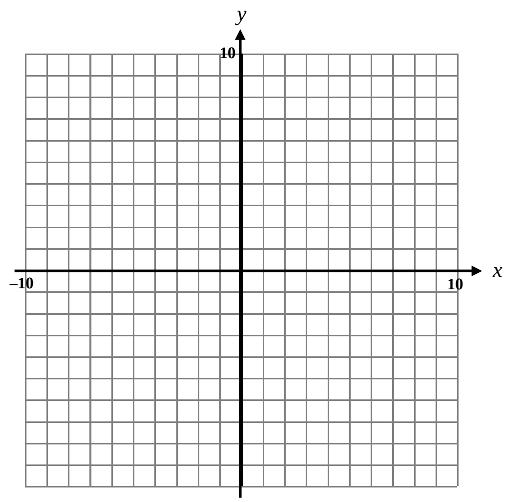
c) ____

6. $y = \left(\frac{1}{2}x - 1\right)^2 + 3$

a) ________________________

____

b)



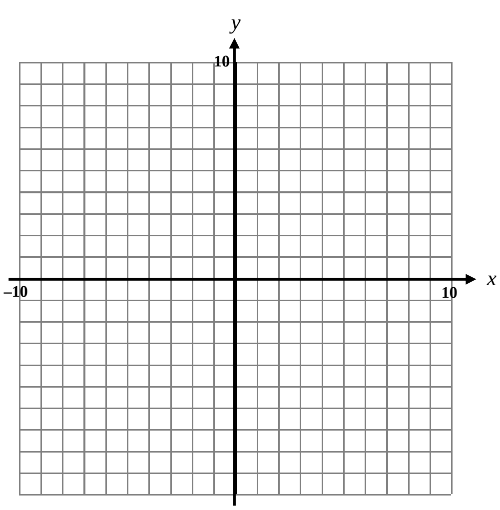
c) ____

7. $y = \sqrt{6 - 3x}$

a) ________________________

____

b)



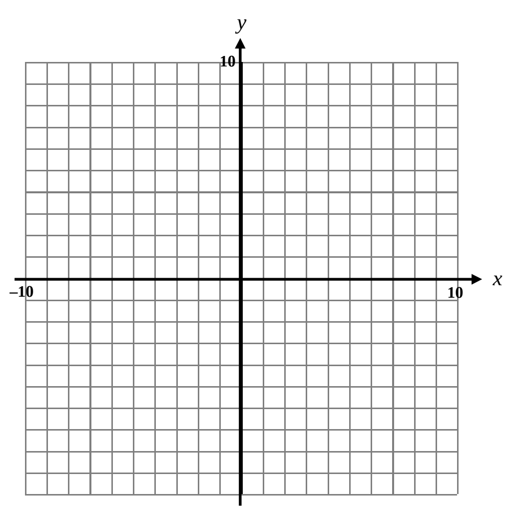
c) ____

8. $y = \dfrac{4}{3 - x} - 2$

a) ________________________

____

b)



c) ____

9. Write an expression using $f(x)$ that shows a <u>reflection</u> of $f(x)$ across the a) the x-axis … and … b) the y-axis.

a) ____________________________ b) ____________________________

For questions 10 and 11, transform the given function by …

a) …a vertical stretch by a factor of 2 **b) …a reflection over the *y*-axis**

c) …a horizontal shrink by a factor of 1/3 **d) …a reflection over the *x*-axis**

10. $f(x) = x^3 - 4x$

a) ____________________________ b) ____________________________

c) ____________________________ d) ____________________________

11. $f(x) = \frac{1}{x+2}$

a) ____________________________ b) ____________________________

c) ____________________________ d) ____________________________

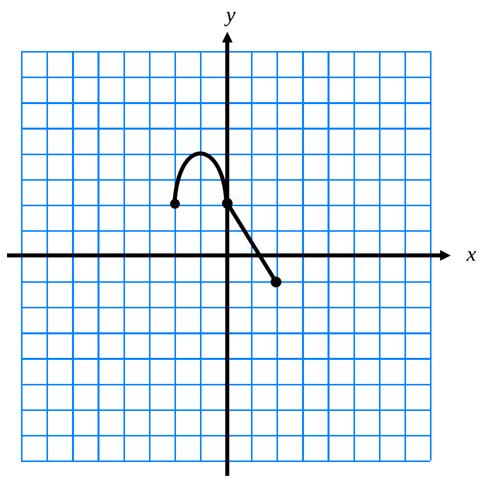
Multiple Choice: For questions 12 and 13, suppose (6, 2) is a point on the graph of $y = f(x)$.

12. Which point must be on the graph of $y = f(-x)$? A) (2, 6) B) (6, –2) C) (–6, 2) D) (–2, 6)

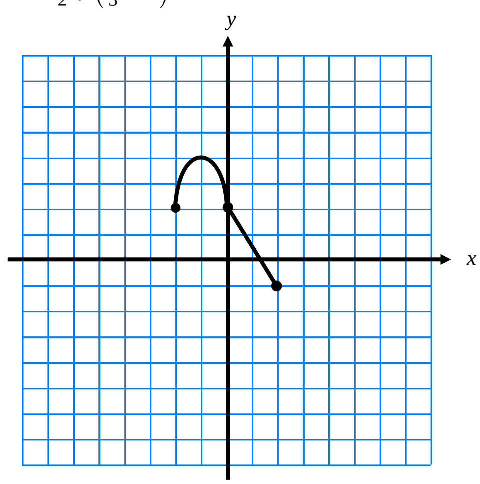
13. Which point must be on the graph of $\frac{1}{2} f(2x)$? A) (12, 4) B) (3, 1) C) (12, 1) D) (3, 4)

For questions 14 and 15, use the graph of $f(x)$ shown to the below to graph each transformation.

14. $-f(2x-6)$

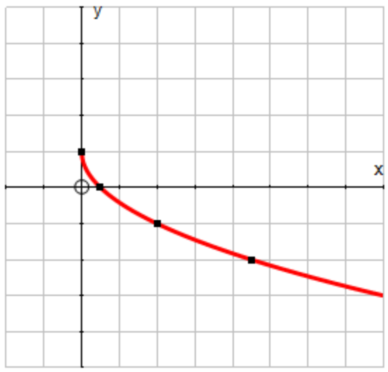


15. $\frac{1}{2} f\left(\frac{x}{3}+2\right)+1$

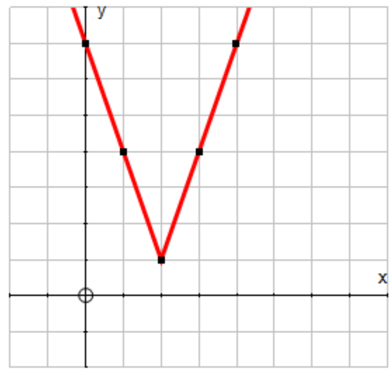


For questions 16 and 17, use the given graph to write the equation of the function shown. Remember to consider the parent function and the transformations that were applied.

16.

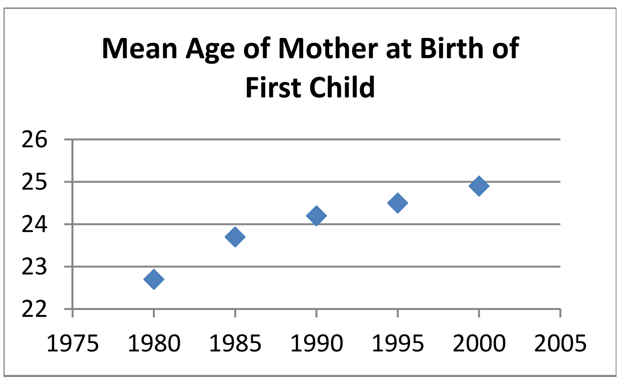


17.

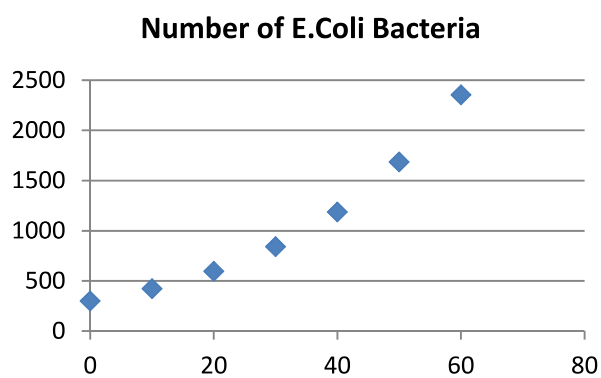


Based on your knowledge of parent functions, use each given data set to identify which parent function fits the data. Explain your choice by using the vocabulary from lesson 1-2.

18.



19.



Pre Calculus
Worksheet 1.4

♬: You have 2 Blocks to Complete this Worksheet

Adding, Subtracting, Multiplying and Dividing Functions

1. Suppose $f(x) = 2x - 1$ and $g(x) = x^2$.

   a) Find $(f+g)(x)$ and state the domain.

   b) Find $(g-f)(x)$ and state the domain.

   c) Find $(fg)(x)$ and state the domain.

2. Find $\left(\frac{f}{g}\right)(x)$ and $\left(\frac{g}{f}\right)(x)$ and state the domain for each using the following functions:

   a) $f(x) = \sqrt{x+3}$ and $g(x) = x^2$

   b) $f(x) = \sqrt{x-2}$ and $g(x) = \sqrt{x+4}$

Composite Functions

3. Find $h(k(x))$ and $k(h(x))$ for the following functions and state the domain for each.

   a) $h(x) = x^2 - 3x + 1$ and $k(x) = \log_5(x)$

   b) $h(x) = \dfrac{3x}{x+7}$ and $k(x) = e^x$

4. You don't always have to create a new function in order to evaluate the function. If I write $h(k(9))$ it means plug 9 into the function *k*, then plug the result into *h*.

   a) Suppose $m(x) = x^2 - 2x + 3$ and $n(x) = 10x + 3$. Evaluate $n(m(-2))$.

   b) Suppose $p(x) = \sqrt{2x+10}$ and $q(x) = \log_2(x)$. Find $p(q(8))$.

In calculus, there is a rule called the chain rule (it is used to take a derivative of a composite function). In order to apply this rule, you must determine what the "inside" and "outside" of a function is. This skill is equivalent to finding the two functions that make up a composite function $f\left(g\left(x\right)\right)$, where g (x) is the "inside" function.

5. Find $f\left(x\right)$ and $g\left(x\right)$ so that each function can be described as $y=f\left(g\left(x\right)\right)$.

a) $y=\sqrt{x^2+5x}$

b) $y=\left|3x-2\right|$

c) $y=e^{\sin x}$

6. If f and g are odd functions, SHOW that the composite function $f\circ g$ is also odd.

7. The surface area S (in square meters) of a hot-air balloon is given by $S\left(r\right)=4\pi r^2$, where r is the radius of the balloon (in meters). If the radius is increasing with time t (in seconds) according to the formula $r\left(t\right)=\frac{2}{3}t^3$, $t\geq 0$, find the surface area of the balloon as a function of the time t.

8. The volume V of a right circular cylinder of height h and radius r is $V=\pi r^2 h$. If the radius is half the height, express the volume V as a function of h.

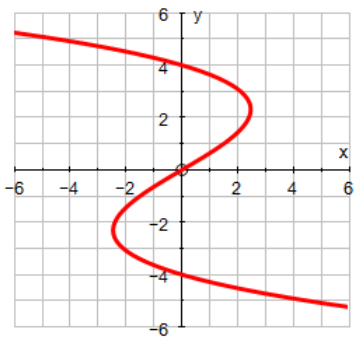
9. The volume V of a right circular cone is $V=\frac{1}{3}\pi r^2 h$. If the radius is one-third of the height, express the volume V as a function of r.

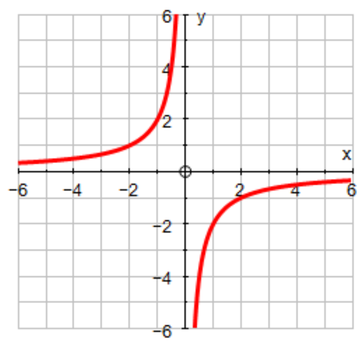
10. The price p of a certain product and the quantity x sold obey the demand equation $p=-\frac{1}{4}x+100$, $0\leq x<400$. Suppose that the cost C of producing x units is $C=\frac{\sqrt{x}}{25}+600$. Assuming that all items produced are sold, find the cost C as a function of the price p.

Inverse Functions

11. What does the Vertical Line Test tell you about a graph? What does the Horizontal Line Test tell you about a graph?

12. For each relation graphed below, determine whether or not the relation represents a function AND whether or not the inverse of the relation represents a function.





13. Which ordered pair is in the *inverse* of the relation given by $x^2y + 5y = 9$?

A (2, 1)
B (–2, 1)
C (–1, 2)
D (2, –1)
E (1, –2)

Finding and Verifying Inverses

14. How do you find an inverse of a function algebraically?

15. How do you prove that two functions are inverses of each other?

16. Find a formula for the inverse function AND prove that the two functions are inverses of each other.

a) $h(x) = 2x + 5$

b) $g(x) = \sqrt[3]{x+5} - 2$

17. Find a formula for the inverse function AND prove that the two functions are inverses of each other if $f(x) = \dfrac{3x+1}{x-4}$.

18. A teacher gives a challenging algebra test. The lowest score is 52 which the teacher decides to scale to 70. The highest score is 88, which the teacher decides to scale to 97.

a) Using the points (52, 70) and (88, 97) find a linear equation that can be used to convert raw scores (original) to scaled scores (new).

b) Find the inverse of the function defined by this linear equation. What does the inverse function do?

19. To convert from x degrees Celsius to y degrees Fahrenheit, we use the formula $y = f(x) = \frac{9}{5}x + 32$. To convert from x degrees Fahrenheit to y degrees Celsius, we use the formula $y = g(x) = \frac{5}{9}(x - 32)$. Show that f and g are inverses.

Pre Calculus
Worksheet 1.6

In order to model a real problem situation with a function, the first step is to define your variables. One useful strategy for problems that “appear” to have too many variables is to define on variable “in terms of” of another variable. This should leave only the independent and dependent variables remaining. Remember, sketches may be helpful.

1. If the length of a rectangle is 1 meter more than three times the width, then write an equation for the perimeter of the rectangle in terms of the width of the rectangle.

2. The height of a right circular cylinder equals its diameter. Write the volume of the cylinder as a function of its radius.

3. One leg of a right triangle is twice as long as the other. Write the length of the hypotenuse as a function of the shorter leg.

4. The base of an isosceles triangle is half as long as the two equal sides. Write the area of the triangle as a function of the length of the base.

Once the equation is set up, with enough additional information, you can solve the problem.

5. Bob installed new windows in his house which were advertised to cut his energy bill by 7.1% per month after installation. If his new energy bill was $315, what was his monthly bill before he installed the new windows?

6. When a number is added to its double and its triple, the sum is 714. Find the three numbers.

7. Ruth is weighing two job offers from the sales departments at two competing companies. One offers a base salary of $35,000 plus a commission of 5% of the gross sales; the other offers a base salary of $28,000 plus 7% commission. What would Ruth’s gross sales need to be in order for the second job offer to be more appealing than the first?

8. Emily’s Decorating Service recommends putting a border around the top of the four walls in a dining room that is 3 feet longer than it is wide. Find the dimensions of the room if the border needed is 54 feet long.

You are now ready to tackle the “mixture” type problems. Remember to define your variables!

9. Coffee worth $3.75 a pound was mixed with coffee worth $4.35 a pound to produce a blend worth $4.11 a pound. How much of each kind of coffee was used to produce 40 pounds of the blended coffee?

10. A 3% solution of sulfuric acid was mixed with an 18% solution of sulfuric acid to produce an 8% solution. How much 3% solution and how much 18% solution were used to produce 15 L of the 8% solution?

11. Reggie invests $12000, part at 7% annual interest and the rest at 8.5% annual interest. How much money was invested at each rate if Reggie's total annual interest was $900?

12. Josh's Pre Calculus final exam is 25% of his overall semester grade. Josh didn't study and the grade on his final exam was 13.5 points below his current grade. If Josh's overall semester grade ended up to be 88.525%, what was his current grade going into the final exam? What did he score on the final exam? Do you think Josh should have studied?

The final concept in modeling with functions is that one function can be used to determine the maximum or minimum for a certain variable.

13. Suppose a piece of cardboard 100 cm by 80 cm is used to make an open top box by cutting out squares that are x cm by x cm from the corners.

a) Express the volume of the box as a function of the length x of the square that is cut out of each corner.

b) What is the domain of this function?

c) Find the maximum volume of the box that can be made. What are the dimensions of this box and what was the length of x that was cut out of the original cardboard?

14. A 216 square meter rectangular pea patch is to be enclosed by a fence and divided into two equal parts by another fence parallel to the shorter of the sides.

a) What dimensions for the outer rectangle will require the smallest total length of fence?

b) How much fence is needed?