Advanced Algebra

Unit 2 Practice: Absolute Value & Piece-wise Functions

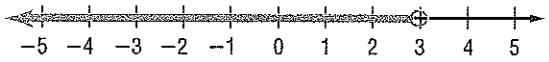
$$y = 2 \cdot |x + 4| - 1$$

$$f(x) = \begin{cases} -4x, if\ x > 2 \\ x - 7, if\ x \leq 2 \end{cases}$$

Check for Understanding

Concept Check

1. **Explain** why it is not necessary to state a division property for inequalities.
2. **Write** an inequality using the $>$ symbol whose solution set is graphed below.



3. **OPEN ENDED** Write an inequality for which the solution set is the empty set.

Guided Practice

Solve each inequality. Describe the solution set using set-builder or interval notation. Then graph the solution set on a number line.

4. $a + 2 < 3.5$
5. $5 \geq 3x$
6. $11 - c \leq 8$
7. $4y + 7 > 31$
8. $2w + 19 < 5$
9. $-0.6p < -9$
10. $\frac{n}{12} + 15 \leq 13$
11. $\frac{5z + 2}{4} < \frac{5z}{4} + 2$

Define a variable and write an inequality for each problem. Then solve.

12. The product of 12 and a number is greater than 36.
13. Three less than twice a number is at most 5.

Application

14. **SCHOOL** The final grade for a class is calculated by taking 75% of the average test score and adding 25% of the score on the final exam. If all scores are out of 100 and a student has a 76 test average, what score does the student need to make on the final exam to have a final grade of at least 80?

Practice and Apply

Homework Help

For Exercises	See Examples
15–40	1–3
41–51	4

Extra Practice See page 829.

Solve each inequality. Describe the solution set using set-builder or interval notation. Then, graph the solution set on a number line.

15. $n + 4 \geq -7$
16. $b - 3 \leq 15$
17. $5x < 35$
18. $\frac{d}{2} > -4$
19. $\frac{g}{-3} \geq -9$
20. $-8p \geq 24$
21. $13 - 4k \leq 27$
22. $14 > 7y - 21$
23. $-27 < 8m + 5$
24. $6b + 11 \geq 15$
25. $2(4t + 9) \leq 18$
26. $90 \geq 5(2r + 6)$
27. $14 - 8n \leq 0$
28. $-4(5w - 8) < 33$
29. $0.02x + 5.58 < 0$
30. $1.5 - 0.25c < 6$
31. $6d + 3 \geq 5d - 2$
32. $9z + 2 > 4z + 15$
33. $2(g + 4) < 3g - 2(g - 5)$
34. $3(a + 4) - 2(3a + 4) \leq 4a - 1$
35. $y < \frac{-y + 2}{9}$
36. $\frac{1 - 4p}{5} < 0.2$
37. $\frac{4x + 2}{6} < \frac{2x + 1}{3}$
38. $12\left(\frac{1}{4} - \frac{n}{3}\right) \leq -6n$

39. **PART-TIME JOB** David earns \$5.60 an hour working at Box Office Videos. Each week, 25% of his total pay is deducted for taxes. If David wants his take-home pay to be at least \$105 a week, solve the inequality $5.6x - 0.25(5.6x) \geq 105$ to determine how many hours he must work.

40. **STATE FAIR** Juan's parents gave him \$35 to spend at the State Fair. He spends \$13.25 for food. If rides at the fair cost \$1.50 each, solve the inequality $1.5n + 13.25 \leq 35$ to determine how many rides he can afford.

Solve each inequality. Graph the solution set on a number line.

8. $y - 3 > 1$ or $y + 2 < 1$
9. $3 < d + 5 < 8$
10. $|a| \geq 5$
11. $|g + 4| \leq 9$
12. $|4k - 8| < 20$
13. $|w| \geq -2$

Application

14. **FLOORING** Deion estimates that he will need between 55 and 60 ceramic tiles to retile his kitchen floor. If each tile costs \$6.25, write and solve a compound inequality to determine what the cost c of the tile could be.

Practice and Apply

Homework Help

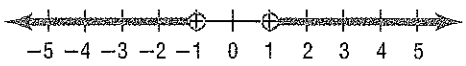
For Exercises	See Examples
15–26, 33–44	3–5
27–32, 51, 52	1, 2
45–50	6

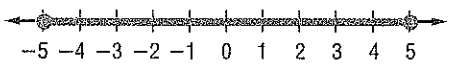
Extra Practice See page 829.

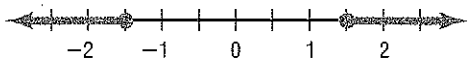
Write an absolute value inequality for each of the following. Then graph the solution set on a number line.

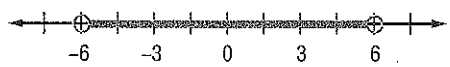
15. all numbers greater than or equal to 5 or less than or equal to -5
16. all numbers less than 7 and greater than -7
17. all numbers between -4 and 4
18. all numbers less than or equal to -6 or greater than or equal to 6
19. all numbers greater than 8 or less than -8
20. all number less than or equal to 1.2 and greater than or equal to -1.2

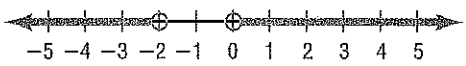
Write an absolute value inequality for each graph.

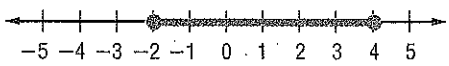
21. 

22. 

23. 

24. 

25. 

26. 

More About . . .

Betta Fish

Adult Male Size: 3 inches
Water pH: 6.8–7.4
Temperature: 75–86°F
Diet: omnivore, prefers live foods
Tank Level: top dweller
Difficulty of Care: easy to intermediate
Life Span: 2–3 years
Source: www.about.com

Solve each inequality. Graph the solution set on a number line.

27. $3p + 1 \leq 7$ or $2p - 9 \geq 7$
28. $9 < 3t + 6 < 15$
29. $-11 < -4x + 5 < 13$
30. $2c - 1 < -5$ or $3c + 2 \geq 5$
31. $-4 < 4f + 24 < 4$
32. $a + 2 > -2$ or $a - 8 < 1$
33. $|g| \leq 9$
34. $|2m| \geq 8$
35. $|3k| < 0$
36. $|-5y| < 35$
37. $|b - 4| > 6$
38. $|6r - 3| < 21$
39. $|3w + 2| \leq 5$
40. $|7x| + 4 < 0$
41. $|n| \geq n$
42. $|n| \leq n$
43. $|2n - 7| \leq 0$
44. $|n - 3| < n$

45. **BETTA FISH** A Siamese Fighting Fish, also known as a Betta fish, is one of the most recognized and colorful fish kept as a pet. Using the information at the left, write a compound inequality to describe the acceptable range of water pH levels for a male Betta.

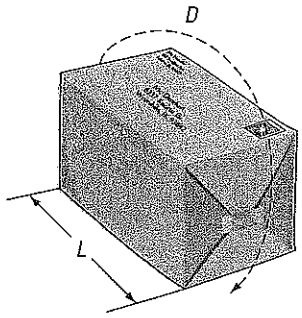
SPEED LIMITS **For Exercises 46 and 47, use the following information.**
On some interstate highways, the maximum speed a car may drive is 65 miles per hour. A tractor-trailer may not drive more than 55 miles per hour. The minimum speed for all vehicles is 45 miles per hour.

46. Write an inequality to represent the allowable speed for a car on an interstate highway.

47. Write an inequality to represent the speed at which a tractor-trailer may travel on an interstate highway.

48. HEALTH *Hypothermia* and *hyperthermia* are similar words but have opposite meanings. Hypothermia is defined as a lowered body temperature. Hyperthermia means an extremely high body temperature. Both conditions are potentially dangerous and occur when a person's body temperature fluctuates by more than 8° from the normal body temperature of 98.6°F. Write and solve an absolute value inequality to describe body temperatures that are considered potentially dangerous.

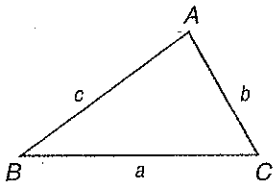
MAIL **For Exercises 49 and 50, use the following information.**
The U.S. Postal Service defines an oversized package as one for which the length L of its longest side plus the distance D around its thickest part is more than 108 inches and less than or equal to 130 inches.



49. Write a compound inequality to describe this situation.

50. If the distance around the thickest part of a package you want to mail is 24 inches, describe the range of lengths that would classify your package as oversized.

GEOMETRY **For Exercises 51 and 52, use the following information.**
The *Triangle Inequality Theorem* states that the sum of the measures of any two sides of a triangle is greater than the measure of the third side.



51. Write three inequalities to express the relationships among the sides of $\triangle ABC$.

52. Write a compound inequality to describe the range of possible measures for side c in terms of a and b. Assume that $a > b > c$. (*Hint*: Solve each inequality you wrote in Exercise 51 for c.)

53. CRITICAL THINKING **Graph each set on a number line.**

a. $-2 < x < 4$ **b.** $x < -1$ or $x > 3$

c. $(-2 < x < 4)$ and $(x < -1$ or $x > 3)$ (*Hint*: This is the intersection of the graphs in part **a** and part **b**.)

d. Solve $3 < |x + 2| \leq 8$. Explain your reasoning and graph the solution set.

54. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How are compound inequalities used in medicine?

Include the following in your answer:

- an explanation as to when to use *and* and when to use *or* when writing a compound inequality,
- an alternative way to write $h \geq 10$ and $h \leq 16$, and
- an example of an acceptable number of hours for this fasting state and a graph to support your answer.

CHECK

$|x + 6| = 3x - 2$

$|4 + 6| \stackrel{?}{=} 3(4) - 2$

$|10| \stackrel{?}{=} 12 - 2$

$10 = 10$ ✓

or

$|x + 6| = 3x - 2$

$|-1 + 6| \stackrel{?}{=} 3(-1) - 2$

$|5| \stackrel{?}{=} -3 - 2$

$5 \neq -5$

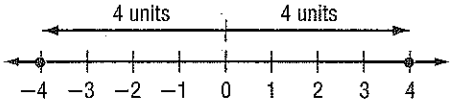
Since $5 \neq -5$, the only solution is 4. Thus, the solution set is {4}.

Check for Understanding

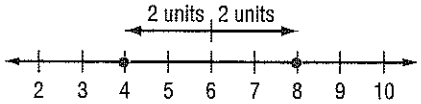
Concept Check

1. **Explain** why if the absolute value of a number is always nonnegative, $|a|$ can equal $-a$.
2. **Write** an absolute value equation for each solution set graphed below.

a.



b.



3. **Determine** whether the following statement is *sometimes*, *always*, or *never* true. Explain.
   For all real numbers a and b, $a \neq 0$, the equation $|ax + b| = 0$ will have one solution.
4. **OPEN ENDED** Write and evaluate an expression with absolute value.

Guided Practice

Evaluate each expression if $a = -4$ and $b = 1.5$.

5. $|a + 12|$
6. $|-6b|$
7. $-|a + 21|$

Solve each equation. Check your solutions.

8. $|x + 4| = 17$
9. $|b + 15| = 3$
10. $|a - 9| = 20$
11. $|y - 2| = 34$
12. $|2w + 3| + 6 = 2$
13. $|c - 2| = 2c - 10$

Application

FOOD **For Exercises 14–16, use the following information.**
A meat thermometer is used to assure that a safe temperature has been reached to destroy bacteria. Most meat thermometers are accurate to within plus or minus 2°F. **Source:** U.S. Department of Agriculture

14. The ham you are baking needs to reach an internal temperature of 160°F. If the thermometer reads 160°F, write an equation to determine the least and greatest temperatures of the meat.
15. Solve the equation you wrote in Exercise 14.
16. To what temperature reading should you bake a ham to ensure that the minimum internal temperature is reached? Explain.

Practice and Apply

Evaluate each expression if $a = -5$, $b = 6$, and $c = 2.8$.

17. $|-3a|$
18. $|-4b|$
19. $|a + 5|$
20. $|2 - b|$
21. $|2b - 15|$
22. $|4a + 7|$
23. $-|18 - 5c|$
24. $-|c - a|$
25. $6 - |3c + 7|$
26. $9 - |-2b + 8|$
27. $3|a - 10| + |2a|$
28. $|a - b| - |10c - a|$

Homework Help

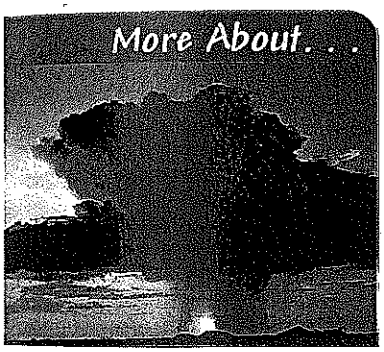
For Exercises	See Examples
17–28	1
29–49	2–4

Extra Practice See page 829.

Solve each equation. Check your solutions.

29. $|x - 25| = 17$
30. $|y + 9| = 21$
31. $|a + 12| = 33$
32. $2|b + 4| = 48$
33. $8|w - 7| = 72$
34. $|3x + 5| = 11$
35. $|2z - 3| = 0$
36. $|6c - 1| = -2$
37. $7|4x - 13| = 35$
38. $-3|2n + 5| = -9$
39. $-12|9x + 1| = 144$
40. $|5x + 9| + 6 = 1$
41. $|a - 3| - 14 = -6$
42. $3|p - 5| = 2p$
43. $3|2a + 7| = 3a + 12$
44. $|3x - 7| - 5 = -3$
45. $4|3t + 8| = 16t$
46. $|15 + m| = -2m + 3$

47. **COFFEE** Some say that to brew an excellent cup of coffee, you must have a brewing temperature of 200°F, plus or minus five degrees. Write and solve an equation describing the maximum and minimum brewing temperatures for an excellent cup of coffee.



Meteorology

The troposphere is characterized by the density of its air and an average vertical temperature change of 6°C per kilometer. All weather phenomena occur within the troposphere.

Source: NASA

48. **MANUFACTURING** A machine is used to fill each of several bags with 16 ounces of sugar. After the bags are filled, another machine weighs them. If the bag weighs 0.3 ounce more or less than the desired weight, the bag is rejected. Write an equation to find the heaviest and lightest bag the machine will approve.

49. **METEOROLOGY** The troposphere is the layer of atmosphere closest to Earth. The average upper boundary of the layer is about 13 kilometers above Earth's surface. This height varies with latitude and with the seasons by as much as 5 kilometers. Write and solve an equation describing the maximum and minimum heights of the upper bound of the troposphere.

CRITICAL THINKING For Exercises 50 and 51, determine whether each statement is *sometimes*, *always*, or *never* true. Explain your reasoning.

50. If a and b are real numbers, then $|a + b| = |a| + |b|$.

51. If a, b, and c are real numbers, then $c|a + b| = |ca + cb|$.

52. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can an absolute value equation describe the magnitude of an earthquake?

Include the following in your answer:

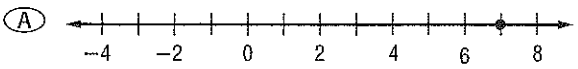
- a verbal and graphical explanation of how $|E - 6.1| = 0.3$ describes the possible extremes in the variation of the earthquake's magnitude, and
- an equation to describe the extremes for a different magnitude.

Standardized Test Practice

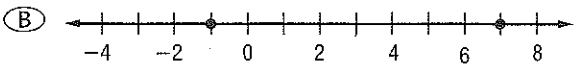
A B C D

53. Which of the graphs below represents the solution set for $|x - 3| - 4 = 0$?

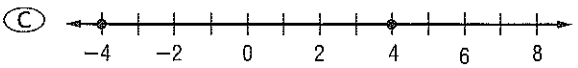
A



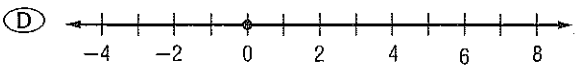
B



C



D



To graph other piecewise functions, examine the inequalities in the definition of the function to determine how much of each piece to include.

Study Tip

Graphs of Piecewise Functions

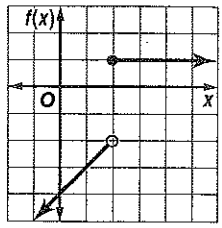
The graphs of each part of a piecewise function may or may not connect. A graph may stop at a given x value and then begin again at a different y value for the same x value.

Example 4 Piecewise Function

Graph $f(x) = \begin{cases} x - 4 \text{ if } x < 2 \\ 1 \text{ if } x \geq 2 \end{cases}$. Identify the domain and range.

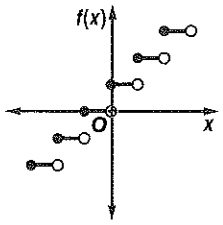
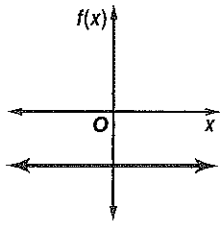
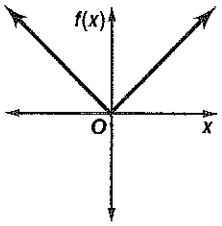
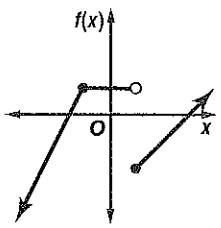
Step 1 Graph the linear function $f(x) = x - 4$ for $x < 2$. Since 2 does not satisfy this inequality, stop with an open circle at $(2, -2)$.

Step 2 Graph the constant function $f(x) = 1$ for $x \geq 2$. Since 2 does satisfy this inequality, begin with a closed circle at $(2, 1)$ and draw a horizontal ray to the right.



The function is defined for all values of x, so the domain is all real numbers. The values that are y-coordinates of points on the graph are 1 and all real numbers less than -2, so the range is $\{y \mid y < -2 \text{ or } y = 1\}$.

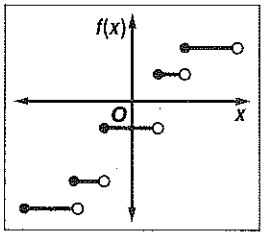
Concept Summary Special Functions

Step Function	Constant Function	Absolute Value Function	Piecewise Function
			
horizontal segments and rays	horizontal line	V-shape	different rays, segments, and curves

Example 5 Identify Functions

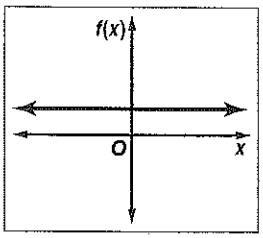
Determine whether each graph represents a step function, a constant function, an absolute value function, or a piecewise function.

a.



The graph has multiple horizontal segments. It represents a step function.

b.



The graph is a horizontal line. It represents a constant function.

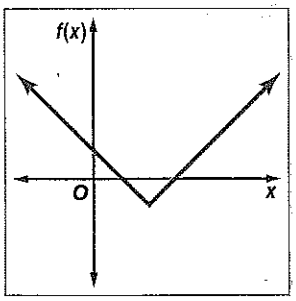
Check for Understanding

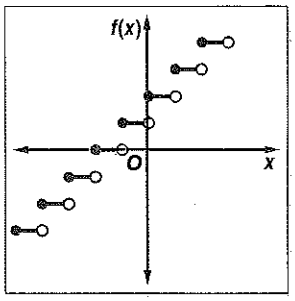
Concept Check

1. **Find a counterexample** to the statement *To find the greatest integer function of x when x is not an integer, round x to the nearest integer.*
2. **Evaluate** $g(4.3)$ if $g(x) = [\![x - 5]\!]$.
3. **OPEN ENDED** Write a function involving absolute value for which $f(-2) = 3$.

Guided Practice

Identify each function as S for step, C for constant, A for absolute value, or P for piecewise.

4. 

5. 

Graph each function. Identify the domain and range.

6. $f(x) = -[\![x]\!]$
7. $g(x) = [\![2x]\!]$
8. $h(x) = |x - 4|$
9. $f(x) = |3x - 2|$
10. $g(x) = \begin{cases} -1 \text{ if } x < 0 \\ -x + 2 \text{ if } x \geq 0 \end{cases}$
11. $h(x) = \begin{cases} x + 3 \text{ if } x \leq -1 \\ 2x \text{ if } x > -1 \end{cases}$

Application

PARKING For Exercises 12–14, use the following information.
A downtown parking lot charges \$2 for the first hour and \$1 for each additional hour or part of an hour.

12. What type of special function models this situation?
13. Draw a graph of a function that represents this situation.
14. Use the graph to find the cost of parking there for $4\frac{1}{2}$ hours.

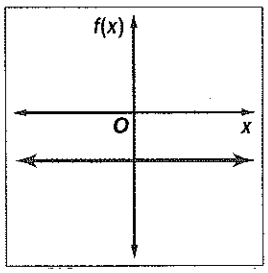
Practice and Apply

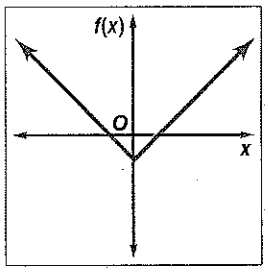
Homework Help

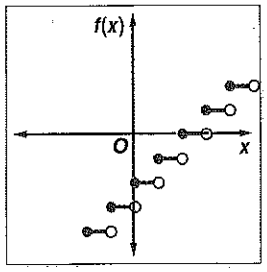
For Exercises	See Examples
15–20	5
21–29	1
30–37, 45–47, 49	3
38–41, 44, 48	2, 4
42, 43	1, 3

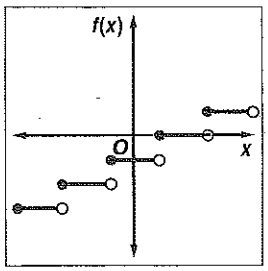
Extra Practice
See page 831.

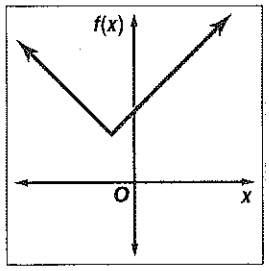
Identify each function as S for step, C for constant, A for absolute value, or P for piecewise.

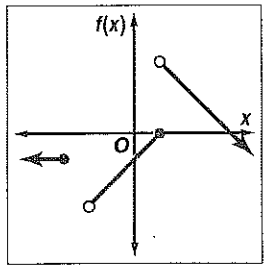
15. 

16. 

17. 

18. 

19. 

20. 

21. **TRANSPORTATION** Bluffton High School chartered buses so the student body could attend the girls' basketball state tournament games. Each bus held a maximum of 60 students. Draw a graph of a step function that shows the relationship between the number of students x who went to the game and the number of buses y that were needed.

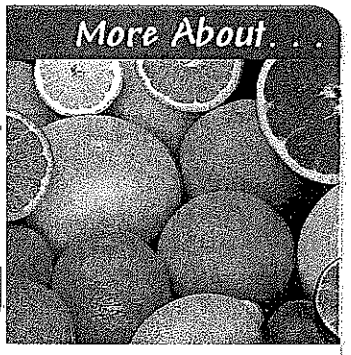
TELEPHONE RATES **For Exercises 22 and 23, use the following information.**
Sarah has a long-distance telephone plan where she pays 10¢ for each minute or part of a minute that she talks, regardless of the time of day.

22. Graph a step function that represents this situation.

23. Sarah made a call to her brother that lasted 9 minutes and 40 seconds. How much did the call cost?

Graph each function. Identify the domain and range.

24. $f(x) = [\![x + 3]\!]$
25. $g(x) = [\![x - 2]\!]$
26. $f(x) = 2[\![x]\!]$
27. $h(x) = -3[\![x]\!]$
28. $g(x) = [\![x]\!] + 3$
29. $f(x) = [\![x]\!] - 1$
30. $f(x) = |2x|$
31. $h(x) = |-x|$
32. $g(x) = |x| + 3$
33. $g(x) = |x| - 4$
34. $h(x) = |x + 3|$
35. $f(x) = |x + 2|$
36. $f(x) = \left|x - \frac{1}{4}\right|$
37. $f(x) = \left|x + \frac{1}{2}\right|$
38. $f(x) = \begin{cases} -x \text{ if } x \leq 3 \\ 2 \text{ if } x > 3 \end{cases}$
39. $h(x) = \begin{cases} -1 \text{ if } x < -2 \\ 1 \text{ if } x > 2 \end{cases}$
40. $f(x) = \begin{cases} x \text{ if } x < -3 \\ 2 \text{ if } -3 \leq x < 1 \\ -2x + 2 \text{ if } x \geq 1 \end{cases}$
41. $g(x) = \begin{cases} -1 \text{ if } x \leq -2 \\ x \text{ if } -2 < x < 2 \\ -x + 1 \text{ if } x \geq 2 \end{cases}$
42. $f(x) = [\![\,|x|\,]\!]$
43. $g(x) = |[\![x]\!]|$

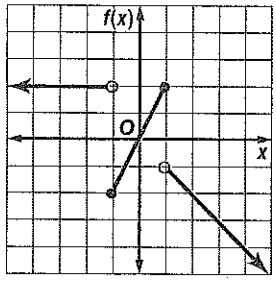


Nutrition

Good sources of vitamin C include citrus fruits and juices, cantaloupe, broccoli, brussels sprouts, potatoes, sweet potatoes, tomatoes, and cabbage.

Source: *The World Almanac*

44. Write the function shown in the graph.



NUTRITION **For Exercises 45–47, use the following information.**
The recommended dietary allowance for vitamin C is 2 micrograms per day.

45. Write an absolute value function for the difference between the number of micrograms of vitamin C you ate today x and the recommended amount.

46. What is an appropriate domain for the function?

47. Use the domain to graph the function.

48. **INSURANCE** According to the terms of Lavon's insurance plan, he must pay the first $300 of his annual medical expenses. The insurance company pays 80% of the rest of his medical expenses. Write a function for how much the insurance company pays if x represents Lavon's annual medical expenses.

49. **CRITICAL THINKING** Graph $|x| + |y| = 3$.

50. WRITING IN MATH Answer the question that was posed at the beginning of the lesson.

How do step functions apply to postage rates?

Include the following in your answer:

- an explanation of why a step function is the best model for this situation, while your gas mileage as a function of time as you drive to the post office cannot be modeled with a step function, and
- a graph of a function that represents the cost of a first-class letter.