

State the excluded values for each.

1)  $\frac{x+3}{2x+6}$

2)  $\frac{20}{20x-32}$

Simplify each expression.

3)  $\frac{n+2}{2n+4}$

4)  $\frac{5v+40}{v+8}$

5)  $\frac{x^2-x-2}{x^2-7x-8}$

6)  $\frac{x^2+11x+24}{x^2-5x-24}$

7)  $\frac{v^2+3v+2}{7v^3+14v^2+7v}$

8)  $\frac{v^3-17v^2+70v}{v^2-3v-70}$

9)  $\frac{x^2-64}{x^2-12x+32}$

10)  $\frac{5a-10}{a^2-4}$

11)  $\frac{3n^2+7n-10}{6n-6}$

12)  $\frac{36-9x}{10x^2-40x}$

## Answers to P3-1

1)  $\{-3\}$

2)  $\left\{\frac{8}{5}\right\}$

3)  $\frac{1}{2}$

4) 5

5)  $\frac{x-2}{x-8}$

6)  $\frac{x+8}{x-8}$

7)  $\frac{v+2}{7v(v+1)}$

8)  $\frac{v(v-7)}{v+7}$

9)  $\frac{x+8}{x-4}$

10)  $\frac{5}{a+2}$

11)  $\frac{3n+10}{6}$

12)  $-\frac{9}{10x}$

**Multiplying/Dividing Rational Expressions**  
P3-2 & P3-3

Name: \_\_\_\_\_

Period: \_\_\_\_\_

For each problem: factor and simplify.

①  $\frac{x^3}{2y^2} \cdot \frac{6y^4}{xy}$

②  $\frac{5xy^2}{4x^2} \cdot \frac{8x^3y}{15y^5}$

③  $\frac{x^2 + 7x + 12}{x - 5} \cdot \frac{2x - 10}{x + 3}$

④  $\frac{x^2 - 3x - 10}{x + 7} \cdot \frac{3x + 21}{6x - 30}$

⑤  $\frac{x - 1}{4xy^3} \cdot \frac{6x^2y}{1 - x}$

⑥  $\frac{13xy^2}{x^2 + 3x - 18} \cdot \frac{x^2 - 9}{26x^4y^2}$

⑦  $\frac{25 - x^2}{14x^3y^8} \cdot \frac{7x^2y}{8x + 40}$

⑧  $\frac{2x^2 + 5x - 7}{x + 4} \cdot \frac{x^2 + 4x}{x^2 - 2x + 1}$

⑨  $\frac{2x + 10}{32 - 8x} \cdot \frac{x^2 - 10x + 24}{x^2 - x - 30}$

⑩  $\frac{12x + 48}{6x - 15} \cdot \frac{4x^2 - 25}{x^2 + 9x + 20}$

Answers:

Ⓒ  $-\frac{3x}{2y^2}$

Ⓕ  $-\frac{x-4}{x+4}$

Ⓓ  $\frac{2x^2}{3y^2}$

Ⓔ  $\frac{x(2x+7)}{x-1}$

Ⓖ  $\frac{4(2x+5)}{x+5}$

Ⓗ  $3x^2y$

Ⓙ  $\frac{x+2}{2}$

Ⓢ  $-\frac{x-5}{16xy^7}$

Ⓛ  $\frac{x+3}{2x^2(x+6)}$

Ⓣ  $-\frac{1}{4}$

Ⓔ  $2(x+4)$

Ⓐ  $\frac{4(2x-5)}{3(x+4)}$

What Do You Call a Message  
Printed on a Lion With Chickenpox?

7	4	5	8	10	8	9	1	3	2	10	9	9	3	2	6	4	10	8
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For each problem: factor and simplify.

$$\textcircled{1} \frac{12m^2n^5}{m+5} \div \frac{3m^3n}{m^2-25}$$

$$\textcircled{2} \frac{n^2-9n+20}{6m^7n^2} \div \frac{5n-20}{10mn^2}$$

$$\textcircled{3} \frac{m^2}{m^2-7m} \div \frac{1}{m^2-4m-21}$$

$$\textcircled{4} \frac{16-2m}{m^2+2m-24} \div \frac{m-8}{3m+18}$$

$$\textcircled{5} \frac{12n-36}{9-n^2} \div \frac{8n^5}{n^2+3n}$$

$$\textcircled{6} \frac{m^2-n^2}{m^2+2mn+n^2} \div \frac{m^2n-mn^2}{7m^2}$$

$$\textcircled{7} \frac{n^2-n-12}{2n^2-15n+18} \div \frac{3n^2-12n}{2n^3-9n^2}$$

$$\textcircled{8} \frac{17mn^3}{m^2+2m-35} \div \frac{34m^8n^4}{m^2+7m}$$

$$\textcircled{9} \frac{4n^3-25n}{3n^2-16n+5} \div (10n+25)$$

Answers:

$$\textcircled{H} 7m(m-n)$$

$$\textcircled{N} -3n^4(n-3)$$

$$\textcircled{T} m(m+3)$$

$$\textcircled{D} -\frac{3}{2n^4}$$

$$\textcircled{U} \frac{4n^4(m-5)}{m}$$

$$\textcircled{R} \frac{1}{2m^4n(m-7)}$$

$$\textcircled{S} \frac{n(2n-9)(n+3)}{3(2n-3)(n-6)}$$

$$\textcircled{I} -\frac{6}{m-4}$$

$$\textcircled{A} \frac{n(2n-5)}{5(3n-1)(n-5)}$$

$$\textcircled{W} \frac{7m}{n(m+n)}$$

$$\textcircled{L} \frac{1}{2m^6n(m-5)}$$

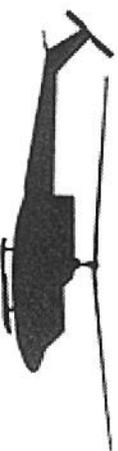
$$\textcircled{E} \frac{n-5}{3m^5}$$

What Happened to the Peanut Who Went Walking Late at Night?

4	3	6	9	7	9	7	7	9	1	8	3	2	5
---	---	---	---	---	---	---	---	---	---	---	---	---	---

## Why Do Helicopters Get Sick So Often?

Express each sum below in simplest form. Find your answer and notice the three letters next to it. Write these letters in the three boxes at the bottom of the page that contain the number of that exercise.



- 1  $\frac{7}{x-3} + \frac{4}{x^2-9}$
- 2  $\frac{x}{x+5} + \frac{7x+10}{x^2+5x}$
- 3  $\frac{x-20}{x^2-4x} + \frac{x}{x-4}$
- 4  $\frac{m}{m+5} + \frac{10m}{m^2-25}$
- 5  $\frac{2}{m+3} + \frac{9}{m^2+8m+15}$
- 6  $\frac{11m}{m^2+3m-28} + \frac{m}{m+7}$
- 7  $\frac{3}{a+2} + \frac{8}{a-5}$
- 8  $\frac{6}{a^2-4} + \frac{2}{a+2} + \frac{5}{a-2}$
- 9  $\frac{2}{a-3} + \frac{7}{a^2+a-12} + \frac{1}{a+4}$

- |   |   |   |   |
|---|---|---|---|
| <b>(COP)</b> $\frac{3m}{m+7}$           | <b>(NTT)</b> $\frac{9a+4}{(a+2)(a-5)}$  | <b>(HEG)</b> $\frac{3}{a-3}$            | <b>(ICK)</b> $\frac{11m+2}{(m+3)(m+5)}$ |
| <b>(BIR)</b> $\frac{m}{m-4}$            | <b>(THE)</b> $\frac{x+2}{x}$            | <b>(HAT)</b> $\frac{4a+9}{(a-3)(a+4)}$  | <b>(EST)</b> $\frac{x+5}{x}$            |
| <b>(DCA)</b> $\frac{7x+25}{(x+3)(x-3)}$ | <b>(RLY)</b> $\frac{7a+12}{(a+2)(a-2)}$ | <b>(ERM)</b> $\frac{2m+19}{(m+3)(m+5)}$ | <b>(TCH)</b> $\frac{11a+1}{(a+2)(a-5)}$ |
| <b>(TES)</b> $\frac{5a+14}{(a+2)(a-2)}$ | <b>(WHI)</b> $\frac{m}{m-5}$            | <b>(ENT)</b> $\frac{x-1}{x}$            | <b>(HEL)</b> $\frac{2m}{m+5}$           |

2	2	2	4	4	4	4	8	8	8	8	6	6	6	6	1	1	1	1	7	7	7	7	3	3	3	3	9	9	9	9	5	5	5	5
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

## P3-5

Simplify each expression.

1)  $\frac{5x}{6y} - \frac{x+2y}{6y}$

2)  $\frac{4y}{5x^3} - \frac{6}{6xy}$

3)  $\frac{4m}{2} - \frac{m-5n}{5m}$

4)  $\frac{3}{2x(x+6)} - \frac{5}{2}$

5)  $\frac{6b}{b+1} - \frac{2b-6}{b-3}$

6)  $\frac{6k}{2k} - \frac{k+4}{2k^2+6k-8}$

7)  $\frac{4}{p-5} - \frac{2}{4p+20}$

8)  $\frac{3x}{5x-6} - \frac{5}{x+5}$

## Answers to P3-5

1)  $\frac{2x-y}{3y}$

2)  $\frac{4y^2-5x^2}{5x^3y}$

3)  $\frac{10m^2-m+5n}{5m}$

4)  $\frac{3-5x^2-30x}{2x(x+6)}$

5)  $\frac{4b-2}{b+1}$

6)  $\frac{6k-7}{2(k-1)}$

7)  $\frac{7p+45}{2(p-5)(p+5)}$

8)  $\frac{3x^2-10x+30}{(5x-6)(x+5)}$

# What Sound Did the Sheep Hear When Her Sister Exploded?



P3-6

Solve each equation and find your answer in the rectangle below. Cross out the box that contains your answer. When you finish, write the letters from the remaining boxes in the spaces at the bottom of the page.

$$\textcircled{1} \quad \frac{2}{x+3} + \frac{3}{x+4} = \frac{7}{x^2 + 7x + 12}$$

$$\textcircled{2} \quad \frac{4}{x-5} + \frac{1}{x+2} = \frac{2x+7}{x^2 - 3x - 10}$$

$$\textcircled{3} \quad \frac{a-30}{a^2 + 4a - 21} = \frac{5}{a+7} - \frac{2}{a-3}$$

$$\textcircled{4} \quad \frac{x}{x+4} = \frac{3}{x-1}$$

$$\textcircled{5} \quad \frac{6}{y+2} + \frac{1}{y-2} = 1$$

$$\textcircled{6} \quad \frac{3}{n} + \frac{2}{n-1} = 2$$

$$\textcircled{7} \quad 2 = \frac{x}{x+3} - \frac{3}{x-5}$$

$$\textcircled{8} \quad \frac{1}{d-7} + \frac{d}{d-2} = \frac{5}{d^2 - 9d + 14}$$

$$\textcircled{9} \quad \frac{x-1}{x+1} - \frac{6}{x-3} = 3$$

YE	SI	CK	SB	AM	SH	OO	FR	KO	MB	IG	UP	AH	ER
6, 1	-5, 2	-1	-9	-3, 1	$-\frac{1}{2}$	2, 8	-7, 3	-2	$\frac{1}{4}, -1$	$\frac{1}{2}, 3$	$\frac{4}{3}$	$\frac{1}{3}, 5$	6, -2

OBJECTIVE 3-h: To solve fractional equations (solving a quadratic equation may be required).

*Determine the equations of any vertical asymptotes and the values of  $x$  for any holes in the graph of each rational function.*

1.  $f(x) = \frac{x^2-25}{x-5}$

2.  $f(x) = \frac{4x-4}{x^2-1}$

3.  $g(x) = \frac{x+2}{x^2-x-6}$

4.  $g(x) = \frac{x+3}{x^2+7x+12}$

5.  $h(x) = \frac{x+2}{x^2-4}$

6.  $h(x) = \frac{x^2-x-6}{(x^2+3x+2)(x^2-4x+3)}$

*Write a rational function in factored form that contains the given characteristics.*

7. Hole:  $x = -4$ , VA:  $x = 5$

8. Hole:  $x = 3, 6$ , VA:  $x = -5$

9. Hole: *none*, VA:  $x = -1, 1$

10. Hole:  $x = -2, -1, 0$ , VA:  $x = 1, 2, 3$

P 3-8

What is the horizontal asymptote for the rational function?

$$\textcircled{1} \quad y = \frac{2x}{x-3}$$

$$\textcircled{2} \quad y = \frac{x-2}{x^2-2x-3}$$

$$\textcircled{3} \quad y = \frac{x^2}{2x-5}$$

$$\textcircled{4} \quad y = \frac{-2x+6}{x-5}$$

$$\textcircled{5} \quad y = \frac{x-1}{x^2+4x+4}$$

$$\textcircled{6} \quad y = \frac{x^2+2x-3}{x-2}$$

$$\textcircled{7} \quad y = \frac{x+2}{2x^2-4}$$

$$\textcircled{8} \quad y = \frac{3x-4}{4x+1}$$

You can solve systems with rational equations using some of same methods you used with linear systems.

**Activity 1**

Follow each direction to solve the system  $\begin{cases} y = \frac{x}{3x-1} \\ y = \frac{1}{x+1} \end{cases}$

1. Set the expressions for y equal to each other.
2. Solve for x.
3. Check your answer by substituting in the original system.

**Activity 2**

Follow each direction to solve the system  $\begin{cases} x-2 = \frac{6}{y} \\ y+1 = x \end{cases}$

4. Solve each equation for y.
5. Set the resulting expressions equal to each other.
6. Solve for x.
7. Check your answer by substituting in the original system.

**Exercises**

Solve each system.

8.  $\begin{cases} \frac{y}{x^2-4x+3} = -2 \\ x-2y = 3 \end{cases}$

9.  $\begin{cases} y = \frac{1}{x} \\ y = \frac{3}{4-x^2} \end{cases}$

10.  $\begin{cases} y = x^2 - 2x - 2 \\ y = \frac{x^2+x-6}{x+3} \end{cases}$

11.  $\begin{cases} y = \frac{x+2}{x^2+3x+2} + 2 \\ y-3 = x \end{cases}$

12. Reasoning It is possible for the graph of a system of rational equations to include a point of intersection that is an extraneous solution? Explain.

Activity 1

①  $\frac{x}{3x-1} = \frac{1}{x+1}$

② LCD:  $(3x-1)(x+1)$

$\frac{x}{3x-1} \cdot (3x-1)(x+1) = \frac{1}{x+1} \cdot (3x-1)(x+1)$

$x(x+1) = 3x-1$

$x^2 + x = 3x - 1$   
 $-3x \quad -3x$

$x^2 - 2x = -1$   
 $+1 \quad +1$

$x^2 - 2x + 1 = 0$

$(x-1)(x-1) = 0$

$x = 1$

③ CHECK  $x = 1$ :  $\frac{1}{3(1)-1} = \frac{1}{1+1}$

ORIGINAL SYSTEM:  $\frac{1}{2} = \frac{1}{2}$  ✓. 😊

$\begin{cases} y = \frac{1}{3(1)-1} = \frac{1}{2} \\ y = \frac{1}{1+1} = \frac{1}{2} \end{cases}$

Solution:  $(1, \frac{1}{2})$

Activity 2

④  $x-2 = \frac{6}{y}$  multiply by y to both sides of eq.

$y(x-2) = y \cdot \frac{6}{y}$

$\frac{y(x-2)}{(x-2)} = \frac{6}{(x-2)}$

$y = \frac{6}{x-2}$

$y+1 = x$   
 $-1 \quad -1$

$y = x-1$

⑤  $\frac{6}{x-2} = x-1$

⑥ LCD:  $(x-2)$

$(x-2) \cdot \frac{6}{x-2} = (x-2)(x-1)$

$6 = (x-2)(x-1)$

$6 = x^2 - 1x - 2x + 2$   
 $-6 \quad -6$

$0 = x^2 - 3x - 4$

$0 = (x-4)(x+1)$

$0 = x-4 \text{ or } 0 = x+1$

$4 = x \quad -1 = x$

⑦  $\begin{cases} y = \frac{6}{x-2} \\ y = x-1 \end{cases}$

when  $x = 4$ :  
 $y = \frac{6}{4-2} = \frac{6}{2} = 3$

$(4, 3)$

when  $x = -1$ :  
 $y = -1-1 = -2$

$(-1, -2)$

Also could substitute into 2<sup>nd</sup> eq.

Exercises

Solve graphically or algebraically.

## HHA Key Concept 3 Review

Name: \_\_\_\_\_

**Problems 1-8:** Simplify each expression

1.  $\frac{2}{7x} + \frac{4}{x^2}$

2.  $\frac{11}{6x} + \frac{5}{2x^2}$

3.  $\frac{2}{x+1} - \frac{3}{x+2} + \frac{8}{x^2+3x+2}$

4.  $\frac{9}{x+4} + \frac{6}{x-4} - \frac{12}{x^2-16}$

5.  $\frac{x^2-49}{x^2-4x-60} \cdot \frac{x^2+10x+24}{4x-28}$

6.  $\frac{x^2+8x-9}{3x-12} \cdot \frac{x^2+5x-36}{x^2+x-2}$

7.  $\frac{\frac{12}{x-4}}{\frac{1}{5} - \frac{3}{x-4}}$

8.  $\frac{4x-20}{x^2+3x-40} \div \frac{8x+16}{x^2-64}$

**Problems 9-16:** Identify any holes, vertical and horizontal asymptotes for the following functions. Then find the x & y intercepts and graph each function.

9.  $f(x) = \frac{(x-1)(x+3)}{(x+3)(x-6)}$

10.  $f(x) = \frac{(x-2)(x+2)}{(x+1)(x-1)}$

11.  $f(x) = \frac{4x-3}{x+1}$

12.  $f(x) = \frac{-3x+2}{x-2}$

13.  $f(x) = \frac{x^2-1}{x^2+5x+4}$

14.  $f(x) = \frac{x^2+5x+6}{x^2-2x-8}$

15.  $f(x) = \frac{3x^2-4x+1}{3x^2+5x-2}$

$$16. f(x) = \frac{6x^2 + 13x - 5}{2x^2 + 9x + 10}$$

**Problems 23-25:** Solve the system of rational equations algebraically or graphically.

$$23. \begin{cases} y = \frac{6}{x+1} \\ y = x \end{cases}$$

**Problems 17-22:** Solve the rational equation and checking for extraneous solutions.

$$17. \frac{2}{x} = \frac{x+7}{x}$$

$$18. \frac{x^2 - 3x - 4}{x+6} = \frac{50}{x+6}$$

$$19. \frac{2}{x+3} = 4 - \frac{2x}{x+3}$$

$$20. \frac{-1}{x-1} = x - \frac{x}{x-1}$$

$$21. \frac{1}{r-2} + \frac{1}{r^2 - 7r + 10} = \frac{6}{r-2}$$

$$22. \frac{7x}{x^2 - x - 12} = \frac{x}{x-4} - \frac{4}{x+3}$$

$$24. \begin{cases} y = \frac{x}{x+2} \\ y = \frac{2x}{x-1} \end{cases}$$

$$25. \begin{cases} xy = 720 \\ x + y = 149 \end{cases}$$