

**P4-1: Simplifying Radicals**

Honors Advanced Algebra

Name: \_\_\_\_\_

Period: \_\_\_\_\_ Date: \_\_\_\_\_

Simplify. Use absolute value signs when necessary.

1)  $\sqrt{24}$

2)  $\sqrt[3]{1000}$

3)  $\sqrt[3]{-162}$

4)  $\sqrt{512}$

5)  $\sqrt[4]{128n^8}$

6)  $\sqrt{98k}$

7)  $\sqrt[5]{224r^7}$

8)  $\sqrt[3]{24m^3}$

9)  $\sqrt{392x^2}$

10)  $\sqrt{512x^2}$

11)  $\sqrt[4]{405x^3y^2}$

12)  $\sqrt[3]{-16a^3b^8}$

13)  $\sqrt[4]{128x^7y^7}$

14)  $\sqrt[3]{16xy}$

15)  $\sqrt[6]{448x^7y^7}$

16)  $\sqrt[3]{56x^5y}$

## ANSWERS

1.  $2\sqrt{6}$
2. 10
3.  $-3\sqrt[3]{6}$
4.  $16\sqrt{2}$
5.  $2n^2\sqrt[4]{8}$
6.  $7\sqrt{2k}$
7.  $2r^5\sqrt{7r^2}$
8.  $2m^3\sqrt[3]{3}$
9.  $14|x|\sqrt{2}$
10.  $16|x|\sqrt{2}$
11.  $3^4\sqrt[4]{5x^3y^2}$
12.  $-2ab^2\sqrt[3]{2b^2}$
13.  $2|x| \cdot |y|\sqrt[4]{8x^3y^3}$
14.  $2^3\sqrt{2xy}$
15.  $2|x| \cdot |y|\sqrt[6]{7xy}$
16.  $2x^3\sqrt[3]{7x^2y}$

**P4-2: Operations with Radical Expressions**

Honors Advanced Algebra

Name: \_\_\_\_\_

Period: \_\_\_\_\_ Date: \_\_\_\_\_

Show your work on a separate sheet of paper.

**Simplify.**

1.  $\sqrt{540}$

2.  $\sqrt[3]{-432}$

3.  $\sqrt[3]{128}$

4.  $-\sqrt[4]{405}$

5.  $\sqrt[3]{-5000}$

6.  $\sqrt[5]{-1215}$

7.  $\sqrt[3]{125t^6w^2}$

8.  $\sqrt[4]{48v^8z^{13}}$

9.  $\sqrt[3]{8g^3h^8}$

10.  $\sqrt{45x^3y^8}$

11.  $\sqrt{\frac{11}{9}}$

12.  $\sqrt[3]{\frac{216}{24}}$

13.  $\sqrt{\frac{1}{128}c^4d^7}$

14.  $\sqrt{\frac{9a^5}{64b^4}}$

15.  $\sqrt[4]{\frac{8}{9a^3}}$

16.  $(3\sqrt{15})(-4\sqrt{45})$

17.  $(2\sqrt{24})(7\sqrt{18})$

18.  $\sqrt{810} + \sqrt{240} - \sqrt{250}$

19.  $6\sqrt{20} + 8\sqrt{5} - 5\sqrt{45}$

20.  $8\sqrt{48} - 6\sqrt{75} + 7\sqrt{80}$

21.  $(3\sqrt{2} + 2\sqrt{3})^2$

22.  $(3 - \sqrt{7})^2$

23.  $(\sqrt{5} - \sqrt{6})(\sqrt{5} + \sqrt{2})$

24.  $(\sqrt{2} + \sqrt{10})(\sqrt{2} - \sqrt{10})$

25.  $(1 + \sqrt{6})(5 - \sqrt{7})$

26.  $(\sqrt{3} + 4\sqrt{7})^2$

27.  $(\sqrt{108} - 6\sqrt{3})^2$

28.  $\frac{\sqrt{3}}{\sqrt{5} - 2}$

29.  $\frac{6}{\sqrt{2} - 1}$

30.  $\frac{5 + \sqrt{3}}{4 + \sqrt{3}}$

31.  $\frac{3 + \sqrt{2}}{2 - \sqrt{2}}$

32.  $\frac{3 + \sqrt{6}}{5 - \sqrt{24}}$

33.  $\frac{3 + \sqrt{x}}{2 - \sqrt{x}}$

34. **BRAKING** The formula  $s = 2\sqrt{5\ell}$  estimates the speed  $s$  in miles per hour of a car when it leaves skid marks  $\ell$  feet long. Use the formula to write a simplified expression for  $s$  if  $\ell = 85$ . Then evaluate  $s$  to the nearest mile per hour.

35. **PYTHAGOREAN THEOREM** The measures of the legs of a right triangle can be represented by the expressions  $6x^2y$  and  $9x^2y$ . Use the Pythagorean Theorem to find a simplified expression for the measure of the hypotenuse.

# P4-2 Solutions

Simplify.

1.  $\sqrt{540} - 6\sqrt{15}$

2.  $\sqrt[3]{-432} - 6\sqrt[3]{2}$

3.  $\sqrt[3]{128} - 4\sqrt[3]{2}$

4.  $-\sqrt[3]{405} - 3\sqrt[3]{5}$

5.  $\sqrt[3]{-5000} - 10\sqrt[3]{5}$

6.  $\sqrt[3]{-1215} - 3\sqrt[3]{5}$

7.  $\sqrt[3]{125t^6w^9} - 5t^2\sqrt[3]{w^3}$

8.  $\sqrt[4]{48a^8b^{12}} - 2v^2\sqrt[3]{3z}$

9.  $\sqrt[3]{8g^3k^8} - 2gk^2\sqrt[3]{k^2}$

10.  $\sqrt{45x^2y^8} - 3xy^4\sqrt{5x}$

11.  $\sqrt{\frac{11}{9} \cdot \frac{\sqrt{11}}{5}}$

12.  $\sqrt[3]{\frac{216}{24}} \cdot \sqrt[3]{9}$

13.  $\sqrt{\frac{1}{128}c^4d^7} - \frac{1}{16}c^2d^3\sqrt{2d}$

14.  $\sqrt{\frac{9a^3}{64b^4} \cdot \frac{3a^2\sqrt{3}}{5b^2}}$

15.  $\sqrt[4]{\frac{8}{9a^3}} \cdot \frac{\sqrt[3]{72a}}{3a}$

16.  $(3\sqrt{15})(-4\sqrt{45})$   
 $-180\sqrt{3}$

17.  $(2\sqrt{24})(7\sqrt{18})$   
 $168\sqrt{3}$

18.  $\sqrt{810} + \sqrt{240} - \sqrt{250}$   
 $4\sqrt{10} + 4\sqrt{15}$

19.  $6\sqrt{20} + 8\sqrt{5} - 5\sqrt{45}$   
 $5\sqrt{5}$

20.  $8\sqrt{48} - 6\sqrt{75} + 7\sqrt{80}$   
 $2\sqrt{3} + 26\sqrt{5}$

21.  $(3\sqrt{2} + 2\sqrt{3})^2$   
 $30 + 12\sqrt{6}$

22.  $(3 - \sqrt{7})^2$   
 $15 - 6\sqrt{7}$

23.  $(\sqrt{5} - \sqrt{6})(\sqrt{5} + \sqrt{2})$   
 $5 + \sqrt{10} - \sqrt{30} - 2\sqrt{3} - 6$

24.  $(\sqrt{2} + \sqrt{10})(\sqrt{2} - \sqrt{10})$   
 $-8$

25.  $(1 + \sqrt{6})(5 - \sqrt{7})$   
 $5 - \sqrt{7} + 5\sqrt{6} - \sqrt{42}$

26.  $(\sqrt{3} + 4\sqrt{7})^2$   
 $115 + 8\sqrt{21}$

27.  $(\sqrt{108} - 6\sqrt{3})^2$   
 $0$

28.  $\frac{\sqrt{3}}{\sqrt{5}-2} \sqrt{15} + 2\sqrt{3}$

29.  $\frac{6}{\sqrt{2}-1} 6\sqrt{2} + 6$

30.  $\frac{5 + \sqrt{3}}{4 + \sqrt{3}} \frac{17 - \sqrt{3}}{13}$

31.  $\frac{3 + \sqrt{2}}{2 - \sqrt{2}} \frac{8 + 5\sqrt{2}}{2}$

32.  $\frac{3 + \sqrt{6}}{5 - \sqrt{24}} 27 + 11\sqrt{6}$

33.  $\frac{3 + \sqrt{x}}{2 - \sqrt{x}} \frac{6 + 5\sqrt{x} + x}{4 - x}$

34. **BRAKING** The formula  $s = 2\sqrt{5l}$  estimates the speed  $s$  in miles per hour of a car when it leaves skid marks  $l$  feet long. Use the formula to write a simplified expression for  $s$  if  $l = 85$ . Then evaluate  $s$  to the nearest mile per hour.  $10\sqrt{17}$ ; 41 mi/h

35. **PYTHAGOREAN THEOREM** The measures of the legs of a right triangle can be represented by the expressions  $6x^2y$  and  $9x^2y$ . Use the Pythagorean Theorem to find a simplified expression for the measure of the hypotenuse.  $3x^2y\sqrt{13}$

## Simplifying Rational Exponents

**Simplify.**

1)  $(n^4)^{\frac{3}{2}}$

2)  $(27p^6)^{\frac{5}{3}}$

3)  $(25b^6)^{-1.5}$

4)  $(64m^4)^{\frac{3}{2}}$

5)  $(a^8)^{\frac{3}{2}}$

6)  $(9r^4)^{0.5}$

7)  $(81x^{12})^{1.25}$

8)  $(216r^9)^{\frac{1}{3}}$

**Simplify. Your answer should contain only positive exponents with no fractional exponents in the denominator.**

9)  $2m^2 \cdot 4m^{\frac{3}{2}} \cdot 4m^{-2}$

10)  $3b^{\frac{1}{2}} \cdot b^{\frac{4}{3}}$

11)  $\left(p^{\frac{3}{2}}\right)^{-2}$

12)  $\left(a^{\frac{1}{2}}\right)^{\frac{3}{2}}$

$$13) \frac{2x^{-\frac{7}{4}}}{4x^{\frac{4}{3}}}$$

$$14) \frac{4x^2}{2x^{\frac{1}{2}}}$$

$$15) \frac{3x^{-\frac{1}{2}} \cdot 3x^{\frac{1}{2}} y^{-\frac{1}{3}}}{3y^{-\frac{7}{4}}}$$

$$16) \frac{3y^{\frac{1}{4}}}{4x^{-\frac{2}{3}} y^{\frac{3}{2}} \cdot 3y^{\frac{1}{2}}}$$

$$17) \left(m \cdot m^{-2} n^{\frac{5}{3}}\right)^2$$

$$18) \left(a^{-1} b^{\frac{1}{3}} \cdot a^{-\frac{4}{3}} b^2\right)^2$$

$$19) \left(\frac{x^{\frac{1}{2}} y^{-2}}{yx^{-\frac{7}{4}}}\right)^4$$

$$20) \frac{(x^3 y^2)^{\frac{3}{2}}}{\left(x^{-1} y^{-\frac{2}{3}}\right)^{\frac{1}{4}}}$$

$$21) \frac{\left(x^{-\frac{1}{2}} y^2\right)^{-\frac{5}{4}}}{x^2 y^{\frac{1}{2}}}$$

$$22) \frac{\left(x^{-\frac{1}{2}} y^4\right)^{\frac{1}{4}}}{x^{\frac{2}{3}} y^{\frac{3}{2}} \cdot x^{-\frac{3}{2}} y^{\frac{1}{2}}}$$

## Simplifying Rational Exponents

**Simplify.**

1)  $(n^4)^{\frac{3}{2}}$   
 $n^6$

2)  $(27p^6)^{\frac{5}{3}}$   
 $243p^{10}$

3)  $(25b^6)^{-1.5}$   
 $\frac{1}{125b^9}$

4)  $(64m^4)^{\frac{3}{2}}$   
 $512m^6$

5)  $(a^8)^{\frac{3}{2}}$   
 $a^{12}$

6)  $(9r^4)^{0.5}$   
 $3r^2$

7)  $(81x^{12})^{1.25}$   
 $243x^{15}$

8)  $(216r^9)^{\frac{1}{3}}$   
 $6r^3$

**Simplify. Your answer should contain only positive exponents with no fractional exponents in the denominator.**

9)  $2m^2 \cdot 4m^{\frac{3}{2}} \cdot 4m^{-2}$   
 $32m^{\frac{3}{2}}$

10)  $3b^{\frac{1}{2}} \cdot b^{\frac{4}{3}}$   
 $3b^{\frac{11}{6}}$

11)  $\left(p^{\frac{3}{2}}\right)^{-2}$   
 $\frac{1}{p^3}$

12)  $\left(a^{\frac{1}{2}}\right)^{\frac{3}{2}}$   
 $a^{\frac{3}{4}}$

$$13) \frac{2x^{-\frac{7}{4}}}{4x^{\frac{4}{3}}}$$

$$\frac{x^{\frac{11}{12}}}{2x^4}$$

$$14) \frac{4x^2}{2x^{\frac{1}{2}}}$$

$$2x^{\frac{3}{2}}$$

$$15) \frac{3x^{-\frac{1}{2}} \cdot 3x^{\frac{1}{2}} y^{-\frac{1}{3}}}{3y^{-\frac{7}{4}}}$$

$$3y^{\frac{17}{12}}$$

$$16) \frac{3y^{\frac{1}{4}}}{4x^{\frac{2}{3}} y^{\frac{3}{2}} \cdot 3y^{\frac{1}{2}}}$$

$$\frac{x^{\frac{2}{3}} y^{\frac{1}{4}}}{4y^2}$$

$$17) \left(m \cdot m^{-2} n^{\frac{5}{3}}\right)^2$$

$$\frac{n^{\frac{10}{3}}}{m^2}$$

$$18) \left(a^{-1} b^{\frac{1}{3}} \cdot a^{-\frac{4}{3}} b^2\right)^2$$

$$\frac{a^{\frac{1}{3}} b^{\frac{14}{3}}}{a^5}$$

$$19) \left(\frac{x^{\frac{1}{2}} y^{-2}}{yx^{\frac{7}{4}}}\right)^4$$

$$\frac{x^{-9}}{y^{12}}$$

$$20) \frac{(x^3 y^2)^{\frac{3}{2}}}{(x^{-1} y^{\frac{2}{3}})^4}$$

$$\frac{y^{\frac{19}{6}} x^{\frac{19}{4}}}{y^6 x^4}$$

$$21) \frac{\left(x^{-\frac{1}{2}} y^2\right)^{-\frac{5}{4}}}{x^2 y^{\frac{1}{2}}}$$

$$\frac{x^{\frac{5}{8}}}{y^3 x^{\frac{1}{2}}}$$

$$22) \frac{\left(x^{-\frac{1}{2}} y^4\right)^{\frac{1}{4}}}{x^{\frac{2}{3}} y^{\frac{3}{2}} \cdot x^{-\frac{3}{2}} y^{\frac{1}{2}}}$$

$$\frac{x^{\frac{17}{24}}}{y}$$



## Solving Radical Equations

Solve each equation. Remember to check for extraneous solutions.

1)  $\sqrt{4x - 7} = 5$

2)  $7 = \sqrt{7 - 6x}$

3)  $5 = \sqrt{5n}$

4)  $x = \sqrt{90 - x}$

5)  $5 = \sqrt{3a + 40}$

6)  $\sqrt{-4 - 2n} = 4$

7)  $(15 - a)^{\frac{1}{2}} = (2a - 3)^{\frac{1}{2}}$

8)  $4 = 2(3x + 7)^{\frac{1}{2}}$

## Answers to Solving Radical Equations

1)  $\{8\}$   
5)  $\{-5\}$

2)  $\{-7\}$   
6)  $\{-10\}$

3)  $\{5\}$   
7)  $\{6\}$

4)  $\{9\}$   
8)  $\{-1\}$

P4-5 Radicals: Domain & Range

Name: \_\_\_\_\_

Use a graphing calculator to sketch a graph of the given function to determine the domain and range of the function. Then describe the transformation compared to the parent function  $y = \sqrt{x}$ .

1) $y = \sqrt{x + 4}$ Domain: Range: Transformation:	2) $y = \sqrt{x - 5}$ Domain: Range: Transformation:
3) $y = \sqrt{x} + 3$ Domain: Range: Transformation:	4) $y = \sqrt{x} - 2$ Domain: Range: Transformation:
5) $y = \sqrt{x + 1} + 2$ Domain: Range: Transformation:	6) $y = \sqrt{x - 3} - 6$ Domain: Range: Transformation:

**Investigate**

These functions have an unexpected look. Make sure to use your graphing calculator to assist you as you discover their domain and ranges.

7) $y = \sqrt{3 - x}$ Domain: Range:	8) $y = \sqrt{2 - x}$ Domain: Range:
9) $y = \sqrt{2 - x} + 1$ Domain: Range:	10) $y = -\sqrt{x} - 3$ Domain: Range:
11) $y = -\sqrt{4 - x} + 2$ Domain: Range:	12) $y = -\sqrt{2 - x} - 1$ Domain: Range:

P4-5 Radicals: Domain & Range

Name: \_\_\_\_\_

Use a graphing calculator to sketch the graph of the given function and determine its domain and range. Then describe the transformation compared to the parent function  $y = \sqrt{x}$ .

<p>1) <math>y = \sqrt{x+4}</math>                      Domain: <math>[-4, \infty)</math>                      Range: <math>[0, \infty)</math>                      Transformation: Translation 4 units left</p>	<p>2) <math>y = \sqrt{x-5}</math>                      Domain: <math>[5, \infty)</math>                      Range: <math>[0, \infty)</math>                      Transformation: Translation 5 units right</p>
<p>3) <math>y = \sqrt{x} + 3</math>                      Domain: <math>[0, \infty)</math>                      Range: <math>[3, \infty)</math>                      Transformation: Translation 3 units up</p>	<p>4) <math>y = \sqrt{x} - 2</math>                      Domain: <math>[0, \infty)</math>                      Range: <math>[-2, \infty)</math>                      Transformation: Translation 2 units down</p>
<p>5) <math>y = \sqrt{x+1} + 2</math>                      Domain: <math>[-1, \infty)</math>                      Range: <math>[2, \infty)</math>                      Transformation: Translation 1 unit left and 2 units up</p>	<p>6) <math>y = \sqrt{x-3} - 6</math>                      Domain: <math>[3, \infty)</math>                      Range: <math>[-6, \infty)</math>                      Transformation: Translation 3 units right and 6 units down</p>

Investigate

These functions have an unexpected look. Make sure to use your graphing calculator to assist you in discovering their domains and ranges.

<p>7) <math>y = \sqrt{3-x}</math>                      Domain: <math>(-\infty, 3]</math>                      Range: <math>[0, \infty)</math></p> <p><math>(3-x \geq 0)</math>  <math>3 \geq x</math>  </p>	<p>8) <math>y = \sqrt{2-x}</math>                      Domain: <math>(-\infty, 2]</math>                      Range: <math>[0, \infty)</math></p> <p><math>(2-x \geq 0)</math>  <math>2 \geq x</math>  </p>
<p>9) <math>y = \sqrt{2-x} + 1</math>                      Domain: <math>(-\infty, 2]</math>                      Range: <math>[1, \infty)</math></p> <p><math>(2-x)</math>  </p>	<p>10) <math>y = -\sqrt{x} - 3</math>                      Domain: <math>[0, \infty)</math>                      Range: <math>(-\infty, -3]</math></p> <p><math>(x \geq 0)</math>  </p>
<p>11) <math>y = -\sqrt{4-x} + 2</math>                      Domain: <math>(-\infty, 4]</math>                      Range: <math>(-\infty, 2]</math></p> <p><math>(4-x \geq 0)</math>  <math>4 \geq x</math>  </p>	<p>12) <math>y = -\sqrt{2-x} - 1</math>                      Domain: <math>(-\infty, 2]</math>                      Range: <math>(-\infty, -1]</math></p> <p><math>(2-x \geq 0)</math>  <math>2 \geq x</math>  </p>

Name: \_\_\_\_\_

Honors Advanced Algebra

Key Concept 4 Review

Period: \_\_\_\_\_

Rewrite using the properties of exponents.

1)  $n^{\frac{2}{9}} \cdot n^{\frac{5}{9}} \cdot n^{\frac{1}{9}}$

2)  $a^{\frac{5}{4}} \cdot a^{\frac{3}{10}} \cdot a^{\frac{2}{5}}$

3)  $\left(\frac{8w^{12}}{343}\right)^{\frac{1}{3}}$

Switch forms. (radical  $\leftrightarrow$  exponential)

Simplify.

4)  $\sqrt{x^3}$

5)  $18^{\frac{2}{3}}$

6)  $\sqrt{128x^{14}y^9}$

7)  $\sqrt[4]{16a^{20}b^{14}}$

What rational exponent must equal "y" for each equation to be true.

Simplify. Is the answer rational or irrational?

8)  $\sqrt[3]{\sqrt{x^7}} = x^y$

9)  $\sqrt[3]{(x-1)^{24}} = (x-1)^y$

10)  $-4\sqrt{10} - 13\sqrt{10}$

11)  $-3\sqrt{2} * 4\sqrt{32}$

Solve the equation. Check for extraneous solutions.

12)  $\sqrt[4]{3x^2 - 48} = \sqrt[4]{2x^2 + 2x}$

13)  $(x+1)^{\frac{3}{2}} - 2 = 25$

22) The population of a small town can be modeled by the function  $P(x) = 15,750\sqrt[3]{x - 1999}$ , where 'x' is the year and 2000 is the first year that applies to this model. Answer these questions based upon the model:

A) Rewrite the model  $P(x) = 15,750\sqrt[3]{x - 1999}$  in rational exponent form.

B) State the domain and range, in interval notation, of the model.

C) Based on the model, what will be the population of the small town in 2030?

D) In what year would the population of the small town reach 50,000?

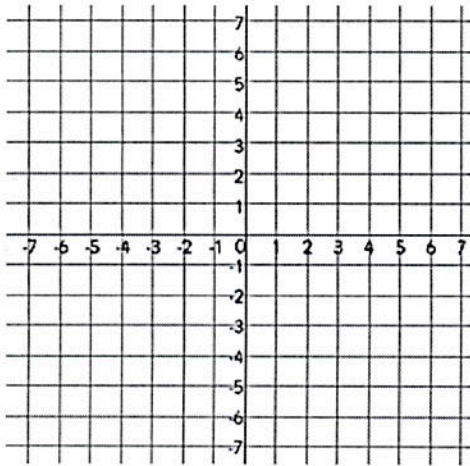
Solve the equation. Check for extraneous solutions.

14)  $2 \cdot \sqrt[3]{4x+16} + 21 = 29$

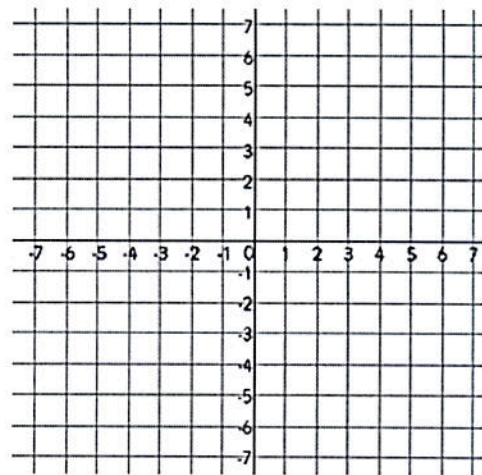
15)  $x - 8 = \sqrt{-25x - 2}$

Graph the function and state the domain and range.

16)  $a(x) = -\sqrt{x+2} + 5$



17)  $b(x) = \frac{1}{2}\sqrt{x-2} - 3$



Describe the transformation from f(x) to g(x).

18)  $f(x) = \sqrt{x+2} - 3$

$g(x) = \sqrt{x} + 1$

19)  $f(x) = \sqrt{x-2}$

$g(x) = \frac{1}{3}\sqrt{x+8} - 5$

Determine the function that matches each graph. State the domain and range too.

