Practice SWIG   Name: _____Key_____

4.A. Understand the relationships between exponential functions and their graphs.

1. What is the range of $y = 4(0.5)^x - 2$?

Range: $\{y \mid y > -2\}$. In other words, $y > -2$, any number bigger than -2.

2. Write an example of an exponential decay function.
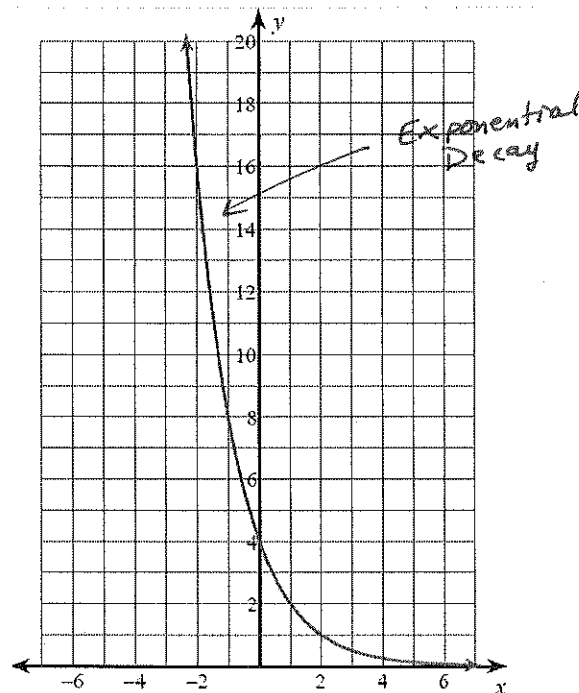
Ex: $y = 2(0.75)^x$ since 0.75 is less than 1.

3. Which type of function is represented in the graph?

a) Exponential growth
b) Exponential decay ⟵ circled
c) neither

Exponential Decay

4. What is the equation of the graph?

a) $y = 4^x$

b) $y = \dfrac{1}{2}^x$

c) $y = \dfrac{1}{2}(4)^x$

d) $y = 4\left(\dfrac{1}{2}\right)^x$ ⟵ circled

e) $y = 2\left(\dfrac{1}{4}\right)^x$

Remember:

$y = a(b)^x$

↓ this number is where the graph will cross the y-axis.

Use the graph above for #3 and 4

4.B. Model and evaluate applications involving exponential growth and decay.

growth

5. The population of Cicero is 35,000 and <u>increasing at</u> the rate of 3% per year. Write a growth model to represent this situation.

Growth Model: $y = a(1+r)^x$

$a = 35,000$

$r = 3\% = 0.03$

$y = 35,000(1+0.03)^x$

$y = 35,000(1.03)^x$

Beginning number →

6. The tiger population beginning in 2003 at an African Wildlife Park can be represented by the model of $y=4500(.94)^t$.

a. How many lions are in the park in 2003?

$$\boxed{4,500}$$

b. Based on the model, are the number of lions increasing or decreasing?

$\boxed{\text{Decreasing}}$ because $.94 < 1$, so this is decay.

c. What percent of the lion population grows/decays each year?

$y = a(1-r)^x$ is the decay model. So $1 - .94 = .6$. $\boxed{\therefore 6\%}$

You deposit $5500 in an account that pays 5% interest per year. How much will be in the account at the end of <u>five years</u> if the bank:

7. compounds <u>continuously</u> $\longrightarrow$ use $A = Pe^{rt}$

$A = Pe^{rt}$ $P = 5,500$, $r = 0.05$, $t = 5$

$= 5,500e^{0.05 \cdot 5}$

$\boxed{\approx \$7,062.14}$

Compounding Interest
(use in #7 and #8)
$A = Pe^{rt}$
$A = P\left(1+\dfrac{r}{n}\right)^{nt}$

8. compounds quarterly

Use $A = P\left(1 + \frac{r}{n}\right)^{nt}$ $P = 5,500$
$r = 0.05$
$= 5,500\left(1 + \frac{0.05}{4}\right)^{4 \cdot 5}$ $n = $ Quarterly $= 4$
$t = 5$ yrs
$\boxed{\approx \$7051.20}$

4.C. Understand how to move between exponential and logarithmic forms.

9. Rewrite the equation in exponential form: $y = \log_3 z$

Same as $\log_3 z = y$ $\boxed{3^y = z}$

10. Rewrite the equation in logarithmic form: $5^x = 625$

$$\boxed{\log_5 625 = x}$$

4.D. Simplify logarithmic expressions.

11. Evaluate: $\log_2 10$ $\boxed{\approx 3.3219}$ to four decimal places

12. Evaluate: $\ln 30$ $\boxed{\approx 3.4012}$ to four decimal places

4.E. Solve exponential and logarithmic equations

12. Solve for x: $3^x = 15$

$$\log_3 15 = x$$

$$\boxed{2.4650 \approx x}$$

(to four decimal places)

13. Solve for x: $3e^x - 1 = 11$

$$\begin{array}{cc} +1 & +1 \end{array}$$

$$\frac{3e^x}{3} = \frac{12}{3}$$

$$e^x = 4$$

$$\log_e 4 = x$$

$$\ln 4 = x$$

$$\boxed{1.3863 \approx x}$$

(to four decimal places)

14. Solve for x: $\log_4(3x+5) = \log_4(2x-1)$

$$\begin{array}{cc} 3x+5 = 2x-1 \\ -2x \qquad -2x \end{array}$$

$$\begin{array}{cc} x+5 = -1 \\ -5 \quad -5 \end{array}$$

$$\boxed{x = -6}$$

15. Solve for x: $\log_2(2x-1) = 3$

$$\left(2^3 = 2 \cdot 2 \cdot 2 \right)$$

$$2^3 = 2x-1$$

$$8 = 2x-1$$

$$\begin{array}{cc} +1 \qquad +1 \end{array}$$

$$\frac{9}{2} = \frac{2x}{2}$$

$$\boxed{4.5 = x}$$

16. You invest $6,000 in an account earning 7.5% annual interest compounded yearly.

How long will it take your money to double?

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

from 6,000 to 12,000

$A = 12,000$
$P = 6,000$
$r = 0.075$
$n =$ compounded yearly
$= 1$

$$12,000 = 6,000\left(1 + \frac{0.075}{1}\right)^{1 \cdot t}$$

$$12,000 = 6,000(1 + 0.075)^t$$

$$\frac{12,000}{6000} = \frac{6,000}{6,000}(1.075)^t$$

$$2 = 1.075^t \longrightarrow$$

$$1.075^t = 2$$

$$\log_{1.075} 2 = t$$

$$\boxed{9.6 \text{ yrs} \approx t}$$

$$t = ?$$

4.F. Condense and expand logarithms using logarithmic properties

17. Expand the expression: $\log 3x^2$

$$\log 3 + \log x^2$$

$$= \boxed{\log 3 + 2 \cdot \log x}$$

Use ① $\log_b(m \cdot n) = \log_b m + \log_b n$
② $p \cdot \log_b m = \log_b m^p$

← Properties

18. Condense the expression: $3\ln x - 2\ln y$

$$3\ln x - 2\ln y$$

$$= \ln x^3 - \ln y^2$$

$$= \ln\left(\frac{x^3}{y^2}\right)$$

Use ① $\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$
② $p \cdot \log_b m = \log_b m^p$