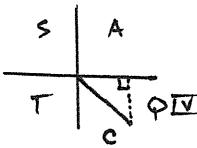


$$\textcircled{3} \quad \sec \theta = 4, \begin{array}{l} \sin \theta < 0 \\ \text{positive} \end{array} \quad \begin{array}{c} \text{negative} \\ \text{Pythagorean Identity} \end{array}$$



$$\textcircled{5} \quad \sin \theta = 0.45. \text{ Find } \cos\left(\frac{\pi}{2} - \theta\right).$$

Cofunction Identity

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

$$\therefore \cos\left(\frac{\pi}{2} - \theta\right) = 0.45$$

$$\begin{aligned} 1 + \tan^2 \theta &= \sec^2 \theta \\ 1 + \tan^2 \theta &= (4)^2 \\ 1 + \tan^2 \theta &= 16 \\ -1 & \\ \tan^2 \theta &= 15 \\ \tan \theta &= \pm \sqrt{15} \end{aligned}$$

Since $\sec \theta$ is positive and $\sin \theta$ is negative $\Rightarrow \tan \theta$ is negative

$$\therefore \tan \theta = -\sqrt{15}$$

$$\begin{aligned} \textcircled{13} \quad \frac{1 + \tan^2 x}{\csc^2 x} &= \frac{\sec^2 x}{\csc^2 x} && \text{pyth. id.} \\ &= \frac{1}{\cos^2 x} && \text{Recip. id.} \\ &= \frac{1}{\sin^2 x} \\ &= \frac{1}{\cos^2 x} \cdot \frac{\sin^2 x}{1} \\ &= \frac{\sin^2 x}{\cos^2 x} && \text{Quotient id.} \\ &= \tan^2 x \end{aligned}$$

$\therefore \frac{1 + \tan^2 x}{\csc^2 x}$ simplifies to $\tan^2 x$.

$$\begin{aligned} \textcircled{17} \quad \sin x \cdot \csc(-x) &= \sin x \{-\csc x\} && \text{Even/odd id.} \\ &= -\sin x \cdot \csc x \\ &= -\sin x \cdot \frac{1}{\sin x} && \text{Recip. id.} \\ &= -\frac{\sin x}{\sin x} \\ &= -1 \end{aligned}$$

$\therefore \sin x \cdot \csc(-x)$ simplifies to -1 .

$$\begin{aligned} \tan \theta &= -\sqrt{15} \\ \cot \theta &= -\frac{1}{\sqrt{15}} \end{aligned}$$

OR

$$\cot \theta = -\frac{\sqrt{15}}{15}$$

$$\textcircled{9} \quad \tan x \cdot \cos x = \frac{\sin x}{\cos x} \cdot \cos x$$

$$= \boxed{\sin x}$$

$\therefore \tan x \cdot \cos x$ simplifies to $\sin x$.

$$\begin{aligned} \textcircled{14} \quad \frac{1 - \cos^2 \theta}{\sin \theta} &= \frac{\sin^2 \theta}{\sin \theta} && \text{Pyth. id.} \\ &= \frac{\sin \theta \cdot \sin \theta}{\sin \theta} \\ &= \boxed{\sin \theta} \end{aligned}$$

$\therefore \frac{1 - \cos^2 \theta}{\sin \theta}$ simplifies to $\sin \theta$.

$$\begin{aligned} \textcircled{15} \quad \cos x - \cos^3 x &= \cos x (1 - \cos^2 x) && \text{Factor out cos x} \\ &= \boxed{\cos x \cdot \sin^2 x} && \text{Pyth. id.} \end{aligned}$$

$\therefore \cos x - \cos^3 x$ simplifies to $\cos x \cdot \sin^2 x$.

$$\begin{aligned} \textcircled{19} \quad \cot(-x) \cdot \cot\left(\frac{\pi}{2} - x\right) &= -\cot x \cdot \tan x && \text{Even/odd + Cofunct id.} \\ &= -\frac{1}{\tan x} \cdot \tan x && \text{Recip. id.} \\ &= -\frac{\tan x}{\tan x} \\ &= -1 \end{aligned}$$

$\therefore \cot(-x) \cdot \cot\left(\frac{\pi}{2} - x\right)$ simplifies to -1 .

$$\textcircled{23} \quad \frac{\tan\left(\frac{\pi}{2} - x\right) \cdot \csc x}{\csc^2 x} = \frac{\cot x \cdot \csc x}{\csc x \cdot \csc x} && \text{Co Funct id.}$$

$$\begin{aligned} &= \frac{\cot x}{\csc x} \\ &= \frac{\cos x}{\sin x} \\ &= \frac{1}{\sin x} && \text{Quotient Recip. id.} \\ &= \frac{\cos x}{\sin x} \\ &= \boxed{\cos x} \end{aligned}$$

$\therefore \frac{\tan\left(\frac{\pi}{2} - x\right) \cdot \csc x}{\csc^2 x}$ simplifies to $\cos x$.

PA 6-2 pg. 410 # 25, 29, 33, 37, 41, 45, 49

$$(25) (\sec^2 x + \csc^2 x) - (\tan^2 x + \cot^2 x) = \sec^2 x + \csc^2 x - \tan^2 x - \cot^2 x$$

Distribute
Rearrange
Pyth. id.

$$= \underline{\sec^2 x - \tan^2 x} + \underline{\csc^2 x - \cot^2 x}$$

$$= 1 + 1$$

$$= \boxed{2}$$

$$(29) \sin x \cdot \cos x \cdot \tan x \cdot \sec x \cdot \csc x$$

$$= \sin x \cdot \cancel{\cos x} \cdot \frac{\sin x}{\cancel{\cos x}} \cdot \frac{1}{\cos x} \cdot \frac{1}{\sin x}$$

Recip. id.
Simplify

$$= \sin x \cdot \frac{1}{\cos x}$$

$$= \frac{\sin x}{\cos x}$$

Quotient id.

$$= \boxed{\tan x}$$

$$(37) \frac{\sec x}{\sin x} - \frac{\sin x}{\cos x} = \frac{\frac{1}{\cos x}}{\frac{\sin x}{1}} - \frac{\sin x}{\cos x}$$

Recip id

$$= \frac{1}{\cos x} \cdot \frac{1}{\sin x} - \frac{\sin x}{\cos x}$$

mult. by recip.

$$= \frac{1}{\cos x} \cdot \frac{1}{\sin x} - \frac{\sin x \cdot \sin x}{\cos x \cdot \sin x}$$

Common denom.

$$= \frac{1}{\cos x \sin x} - \frac{\sin^2 x}{\cos x \sin x}$$

$$= \frac{1 - \sin^2 x}{\cos x \sin x}$$

$$= \frac{\cos^2 x}{\cos x \sin x}$$

Pyth. id.

$$= \frac{\cos x \cdot \cos x}{\cos x \cdot \sin x}$$

$$= \frac{\cos x}{\sin x}$$

Simplify

$$= \boxed{\cot x}$$

Quotient id.

$$(45) 4\tan^2 x - \frac{4}{\cot x} + \sin x \cdot \csc x$$

$$= 4\tan^2 x - \frac{4}{\frac{1}{\tan x}} + \sin x \cdot \frac{1}{\sin x}$$

$$= 4\tan^2 x - 4 \cdot \tan x + 1$$

Let $u = \tan x$. Then, we have:

$$4u^2 - 4u + 1$$

$$= \cancel{4u^2} - \cancel{2u} - \cancel{2u} + 1$$

$$= 2u(2u-1) - 1(2u-1)$$

$$= (2u-1)(2u-1)$$

$$= (2u-1)^2$$

$\therefore (2\tan x - 1)^2$

$$(33) \frac{1}{\sin^2 x} + \frac{\sec^2 x}{\tan^2 x} = \frac{1}{\sin^2 x} + \frac{\frac{1}{\cos^2 x}}{\frac{\sin^2 x}{\cos^2 x}}$$

Recip. id/
Quotient id.
Strategy #1

$$= \frac{1}{\sin^2 x} + \frac{1}{\cos^2 x} \cdot \frac{\cos^2 x}{\sin^2 x}$$

Simplify

$$= \frac{1}{\sin^2 x} + \frac{1}{\sin^2 x}$$

$$= \csc^2 x + \csc^2 x$$

Recipid

$$= \boxed{2\csc^2 x}$$

$$(41) 1 - 2\sin x + (1 - \cos^2 x)$$

$$= 1 - 2\sin x + \sin^2 x$$

$$= \sin^2 x - 2\sin x + 1$$

Let $u = \sin x$.

Then, we have:

$$u^2 - 2u + 1$$

Factor

$$= (u-1)(u-1)$$

$$= (u-1)^2$$

$$\therefore \boxed{(\sin x - 1)^2}$$

$$(49) \frac{\sin^2 x}{1 + \cos x} = \frac{1 - \cos^2 x}{1 + \cos x}$$

Pyth. id.

$$= \frac{(1 + \cos x)(1 - \cos x)}{1 + \cos x}$$

Factor

$$= \boxed{1 - \cos x}$$

Recall: $1 - x^2 = (1-x)(1+x)$
Keep this in mind.

PA 6-3 p. 411 # 51 - 59 odd

$$(51) \quad 2\cos x \sin x - \cos x = 0 \quad (\text{Factor cos x})$$

$$\cos x (2\sin x - 1) = 0$$

$$\cos x = 0 \text{ or } 2\sin x - 1 = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$2\sin x = 1$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$

$$\therefore x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6} \text{ on } [0, 2\pi].$$

$$(53) \quad \tan x \cdot \sin^2 x - \tan x = 0 \quad (\text{Factor tan x})$$

$$\tan x (\sin^2 x - 1) = 0$$

$$\tan x = 0 \text{ or } \sin^2 x - 1 = 0$$

$$\frac{\sin x}{\cos x} = 0$$

$$\sin^2 x = 1$$

$$\sqrt{\sin^2 x} = \pm \sqrt{1}$$

$$x = 0, \pi$$

$$\sin x = \pm 1$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

But note that all values of x must satisfy the original equation.

$$\tan \frac{\pi}{2} = \frac{\sin \frac{\pi}{2}}{\cos \frac{\pi}{2}} = \frac{1}{0}, \text{ undefined}$$

$$\tan \frac{3\pi}{2} \text{ is also undefined.}$$

$$\therefore x = 0, \pi \text{ on } [0, 2\pi].$$

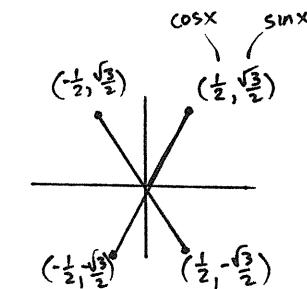
$$(55) \quad \tan^2 x = 3$$

$$\sqrt{\tan^2 x} = \pm \sqrt{3}$$

$$\tan x = \pm \sqrt{3}$$

$$\frac{\sin x}{\cos x} = \pm \sqrt{3}$$

$$\therefore x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$$



$$\text{on } [0, 2\pi]$$

$$(59)$$

$$\sin^2 \theta - 2\sin \theta = 0 \quad (\text{Factor sin } \theta)$$

$$\sin \theta (\sin \theta - 2) = 0$$

$$\sin \theta = 0 \text{ or } \sin \theta - 2 = 0$$

$$\theta = 0, \pm \pi, \pm 2\pi, \dots \quad \sin \theta = 2$$

Range of sine: $[-1, 1]$

\therefore , no solution to equation $\sin \theta = 2$.

\therefore , General solution is:

$$\theta = k\pi, \text{ where } k \text{ is an integer}$$

We can write more concisely as:

$$\theta = k\pi, k \in \mathbb{Z}$$

$$(57) \quad 4\cos^2 x - 4\cos x + 1 = 0$$

Let $u = \cos x$. Then we have:

$$4u^2 - 4u + 1 = 0$$

$$\underline{4u^2 - 2u} \underline{-2u + 1} = 0$$

$$\begin{matrix} 4u^2 \\ -2u \\ \hline -2u + 1 \end{matrix}$$

$$2u(2u - 1) - 1(2u - 1) = 0$$

$$(2u - 1)(2u - 1) = 0$$

$$(2u - 1)^2 = 0$$

$$\sqrt{(2u - 1)^2} = \pm \sqrt{0}$$

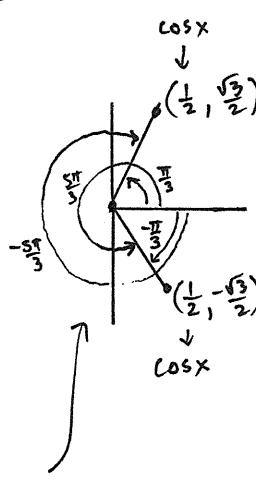
$$2u - 1 = 0$$

$$2u = 1$$

$$u = \frac{1}{2}. \text{ Substitute back}$$

$$\cos x = \frac{1}{2}$$

Note $\cos x$ is positive in QI, QIV



We want all solutions!

Interval $(-\infty, \infty)$

$$x = \pm \frac{\pi}{3}, \pm \frac{5\pi}{3}, \dots$$

\therefore , General solution is:

$$x = \pm \frac{\pi}{3} + 2\pi k, \text{ where } k \text{ is an integer}$$

$$\begin{aligned} \textcircled{11} \quad \text{pf: } (\cos x)(\tan x + \sin x \cdot \cot x) &= \cos x \cdot \tan x + \cos x \cdot \sin x \cdot \cot x \\ &= \cancel{\cos x} \cdot \frac{\sin x}{\cancel{\cos x}} + \cos x \cdot \sin x \cdot \frac{\cos x}{\sin x} \\ &= \sin x + \cos x \cdot \cos x \quad \text{(Simplify)} \\ &= \sin x + \cos^2 x \end{aligned}$$

$$\therefore (\cos x)(\tan x + \sin x \cdot \cot x) = \sin x + \cos^2 x.$$

Distribute
Quotient id.

$$\begin{aligned} \textcircled{15} \quad \text{pf: } \frac{(1-\cos u)(1+\cos u)}{\cos^2 u} &= \frac{1 + \cos u - \cos u - \cos^2 u}{\cos^2 u} \quad \text{Distribute} \\ &= \frac{1 - \cos^2 u}{\cos^2 u} \quad \text{(Simplify)} \\ &= \frac{\sin^2 u}{\cos^2 u} \quad \text{Pyth id.} \\ &= \tan^2 u \quad \text{Quotient id.} \\ \therefore \frac{(1-\cos u)(1+\cos u)}{\cos^2 u} &= \tan^2 u. \end{aligned}$$

$$\begin{aligned} \textcircled{21} \quad \text{Pf: } (\cos t - \sin t)^2 + (\cos t + \sin t)^2 &= (\cos t - \sin t)(\cos t - \sin t) + (\cos t + \sin t)(\cos t + \sin t) \\ &= \cancel{\cos^2 t} - \cancel{\cos t \sin t} - \cancel{\cos t \sin t} + \sin^2 t + \cancel{\cos^2 t} + \cancel{\cos t \sin t} + \cancel{\cos t \sin t} + \sin^2 t \quad \text{Distribute} \\ &= 2\cos^2 t + 2\sin^2 t \quad \text{Combine like terms} \\ &= 2(\cos^2 t + \sin^2 t) \quad \text{Factor} \\ &= 2(1) \quad \text{Pyth. id.} \\ &= 2 \end{aligned}$$

$$\begin{aligned} \textcircled{37} \quad \text{Pf: } \frac{\sin x - \cos x}{\sin x + \cos x} &= \frac{\sin x - \cos x}{\sin x + \cos x} \bullet 1 \\ &= \frac{(\sin x - \cos x)}{(\sin x + \cos x)} \bullet \frac{(\sin x + \cos x)}{(\sin x + \cos x)} \\ &= \frac{\sin^2 x + \cancel{\sin x \cdot \cos x} - \cancel{\sin x \cdot \cos x} - \cos^2 x}{\sin^2 x + \sin x \cdot \cos x + \cancel{\sin x \cdot \cos x} + \cos^2 x} \quad \text{Distribute} \\ &= \frac{\sin^2 x - \cos^2 x}{\sin^2 x + \cos^2 x + 2\sin x \cdot \cos x} \quad \text{Rearrange terms} \quad \text{Simplify} \\ &= \frac{\sin^2 x - (1 - \sin^2 x)}{1 + 2\sin x \cdot \cos x} \quad \text{Pyth. id.} \\ &= \frac{\sin^2 x - 1 + \sin^2 x}{1 + 2\sin x \cdot \cos x} \quad \text{Distribute} \\ &= \frac{2\sin^2 x - 1}{1 + 2\sin x \cdot \cos x} \quad \text{Combine like terms} \\ \therefore \frac{\sin x - \cos x}{\sin x + \cos x} &= \frac{2\sin^2 x - 1}{1 + 2\sin x \cdot \cos x} \end{aligned}$$

(39) PF: $\frac{\sin t}{1-\cos t} + \frac{1+\cos t}{\sin t} = \frac{\sin t}{(1-\cos t)} \cdot \frac{\sin t}{\sin t} + \frac{(1+\cos t)}{\sin t} \cdot \frac{(1-\cos t)}{(1-\cos t)}$ (Common denom)

$$= \frac{\sin^2 t + 1 - \cos t + \cos t - \cos^2 t}{\sin t (1-\cos t)} \quad (\text{Distribute})$$

$$= \frac{(\sin^2 t) - \cos^2 t + 1}{\sin t (1-\cos t)} \quad (\text{Rearrange & simplify})$$

$$= \frac{(1 - \cos^2 t) - \cos^2 t + 1}{\sin t (1-\cos t)} \quad (\text{Pyth. id})$$

$$= \frac{2 - 2 \cos^2 t}{\sin t (1-\cos t)} \quad (\text{Simplify})$$

$$= \frac{2(1 - \cos^2 t)}{\sin t (1-\cos t)} \quad (\text{Factor})$$

$$= \frac{2(1 + \cos t)(1 - \cos t)}{\sin t (1-\cos t)} \quad \downarrow$$

$$= \frac{2(1 + \cos t)}{\sin t} \quad (\text{Simplify})$$

$$\therefore \frac{\sin t}{1-\cos t} + \frac{1+\cos t}{\sin t} = \frac{2(1 + \cos t)}{\sin t}$$

(53) PF: $(1 + \sec x)(1 - \cos x) = 1 - \cos x + \sec x - \sec x \cdot \cos x$ (Distribute)

$$= 1 - \cos x + \frac{1}{\cos x} - \frac{1}{\cos x} \cdot \cos x \quad (\text{Recip. id})$$

$$= 1 - \cos x + \frac{1}{\cos x} - 1 \quad (\text{Simplify})$$

$$= -\cos x + \frac{1}{\cos x} \quad \downarrow$$

$$= -\frac{\cos x}{1} \cdot \frac{\cos x}{\cos x} + \frac{1}{\cos x} \quad (\text{Common denominator})$$

$$= -\frac{\cos^2 x}{\cos x} + \frac{1}{\cos x} \quad (\text{Add numerator})$$

$$= \frac{-\cos^2 x + 1}{\cos x}$$

$$= \frac{1 - \cos^2 x}{\cos x} \quad (\text{Rearrange})$$

$$= \frac{\sin^2 x}{\cos x} \quad (\text{Pyth. id})$$

$$= \frac{\sin x \cdot \sin x}{\cos x} \quad \boxed{\therefore, (d)}$$

$$= \frac{\sin x}{\cos x} \cdot \sin x$$

$$\therefore (1 + \sec x)(1 - \cos x) = \tan x \cdot \sin x = \tan x \cdot \sin x$$

(55) Pf:
$$\begin{aligned} \frac{1}{1+\sin x} + \frac{1}{1-\sin x} &= \frac{1}{(1+\sin x)} \cdot \frac{(1-\sin x)}{(1-\sin x)} + \frac{1}{1-\sin x} \cdot \frac{(1+\sin x)}{(1+\sin x)} \quad (\text{Common denom.}) \\ &= \frac{1 \cdot (1-\sin x) + 1 \cdot (1+\sin x)}{(1+\sin x)(1-\sin x)} \quad (\text{Distribute}) \\ &= \frac{1-\sin x + 1 + \sin x}{1-\sin x + \sin x - \sin^2 x} \\ &= \frac{2}{1 - \sin^2 x} \quad (\text{Pyth id}) \\ &= \frac{2}{\cos^2 x} \\ &= 2 \cdot \frac{1}{\cos^2 x} \quad (\text{Recip. id}) \quad \therefore c \\ &= 2 \cdot \sec^2 x \end{aligned}$$

$$\therefore \frac{1}{1+\sin x} + \frac{1}{1-\sin x} = 2\sec^2 x$$

(57) Pf:
$$\begin{aligned} \frac{1}{\sec x - \tan x} &= \frac{1}{\sec x - \tan x} \cdot 1 \\ &= \frac{1}{(\sec x - \tan x)} \cdot \frac{(\sec x + \tan x)}{(\sec x + \tan x)} \quad (\text{Multiply by } 1) \\ &= \frac{\sec x + \tan x}{\sec^2 x + \sec x \cdot \tan x - \sec x \tan x - \tan^2 x} \quad (\text{Distribute}) \\ &= \frac{\sec x + \tan x}{\underbrace{\sec^2 x - \tan^2 x}_1} \quad (\text{Pyth id}) \\ &= \frac{\sec x + \tan x}{1} \\ &= \sec x + \tan x \end{aligned}$$

$$\therefore \frac{1}{\sec x - \tan x} = \sec x + \tan x.$$

PA6-5 p. 418 # 13, 17, 19, 29, 35, 41, 45, 49, 51

(13) Pf: $(1 - \tan x)^2 = (1 - \tan x)(1 - \tan x)$ (Distribute)
 $= 1 - \tan x - \tan x + \tan^2 x$
 $= 1 - 2\tan x + \tan^2 x$ Pythag. id.
 $= 1 - 2\tan x + \sec^2 x - 1$
 $= \sec^2 x - 2\tan x$ Simplify and rearrange;
 $\therefore (1 - \tan x)^2 = \sec^2 x - 2\tan x.$

(17) Pf: $\frac{\cos^2 x - 1}{\cos x} = \frac{-(\cos^2 x + 1)}{\cos x}$ Factor out "-"
 $= \frac{-(1 - \cos^2 x)}{\cos x}$ Rearrange inside parenthesis
 $= \frac{-\sin^2 x}{\cos x}$ Pythag id
 $= -\frac{\sin x}{\cos x} \cdot \sin x$ $\sin^2 x = \sin x \cdot \sin x$
 $= -\tan x \sin x$ Quotient id
 $\therefore \frac{\cos^2 x - 1}{\cos x} = -\tan x \sin x.$

(19) Pf: $(1 - \sin \beta)(1 + \csc \beta) = 1 + \csc \beta - \sin \beta - \sin \beta \csc \beta$ Distribute
 $= 1 + \csc \beta - \sin \beta - \sin \beta \cdot \frac{1}{\sin \beta}$ Recip. id
 $= 1 + \csc \beta - \sin \beta - 1$
 $= \csc \beta - \sin \beta$ Simplify
 $\therefore (1 - \sin \beta)(1 + \csc \beta) = \csc \beta - \sin \beta.$

(29) Pf: $\cot^2 x - \cos^2 x = \frac{\cos^2 x}{\sin^2 x} - \cos^2 x$ Quotient id
 $= \frac{\cos^2 x}{\sin^2 x} - \frac{\cos^2 x}{1} \cdot \frac{\sin^2 x}{\sin^2 x}$ Common denom.
 $= \frac{\cos^2 x - \cos^2 x \cdot \sin^2 x}{\sin^2 x}$
 $= \frac{\cos^2 x (1 - \sin^2 x)}{\sin^2 x}$ Factor out $\cos^2 x$
 $= \frac{\cos^2 x \cdot \cos^2 x}{\sin^2 x}$ Pythag. id
 $= \cos^2 x \cdot \frac{\cos^2 x}{\sin^2 x}$
 $= \cos^2 x \cot^2 x$

$\therefore \cot^2 x - \cos^2 x = \cos^2 x \cot^2 x.$

$$\textcircled{35} \quad \underline{\text{Pf:}} \quad \frac{\tan x}{\sec x - 1} = \frac{\tan x}{\sec x - 1} \cdot 1$$

$$= \frac{\tan x}{(\sec x - 1)} \cdot \frac{(\sec x + 1)}{(\sec x + 1)}$$

$$= \frac{\tan x (\sec x + 1)}{\sec^2 x + \cancel{\sec x} - \sec x - 1} \quad \text{(Distribute)}$$

$$= \frac{\tan x (\sec x + 1)}{\sec^2 x - 1} \quad \text{(Simplify)}$$

$$= \frac{\tan x (\sec x + 1)}{\tan^2 x} \quad \text{(Pyth. id.)}$$

$$= \frac{\cancel{\tan x} (\sec x + 1)}{\tan x \cdot \tan x} \quad (\tan^2 x = \tan x \cdot \tan x)$$

$$= \frac{\sec x + 1}{\tan x}$$

$$\therefore \frac{\tan x}{\sec x - 1} = \frac{\sec x + 1}{\tan x}.$$

$$\textcircled{41} \quad \underline{\text{Pf:}} \quad \sin^2 x \cdot \cos^3 x = \sin^2 x \cdot \cos^2 x \cdot \cos x \quad (\cos^3 x = \cos^2 x \cdot \cos x)$$

$$= \sin^2 x \cdot (1 - \sin^2 x) \cdot \cos x \quad (\text{Pyth. id.})$$

$$= (\sin^2 x - \sin^4 x)(\cos x) \quad (\text{Distribute})$$

$$\therefore \sin^2 x \cdot \cos^3 x = (\sin^2 x - \sin^4 x)(\cos x).$$

$$\textcircled{45} \quad \underline{\text{Pf:}} \quad \frac{\tan x}{1 - \cot x} + \frac{\cot x}{1 - \tan x} = \frac{\tan x}{1 - \frac{\cos x}{\sin x}} + \frac{\cot x}{1 - \frac{\sin x}{\cos x}} \quad (\text{Quotient id.})$$

$$= \frac{\tan x}{\frac{\sin x}{\sin x} - \frac{\cos x}{\sin x}} + \frac{\cot x}{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}} \quad (\text{Get common denom. in denom.})$$

$$= \frac{\tan x}{\frac{\sin x - \cos x}{\sin x}} + \frac{\cot x}{\frac{\cos x - \sin x}{\cos x}} \quad (\text{Add fract. in denom.})$$

$$= \tan x \cdot \frac{\sin x}{\sin x - \cos x} + \cot x \cdot \frac{\cos x}{\cos x - \sin x} \quad (\text{Multiply by recip. of denom.})$$

$$= \frac{\sin x}{\cos x} \cdot \frac{\sin x}{\sin x - \cos x} + \frac{\cos x}{\sin x} \cdot \frac{\cos x}{\cos x - \sin x} \quad (\text{Quotient id.})$$

$$= \frac{\sin^2 x}{\cos x (\sin x - \cos x)} + \frac{\cos^2 x}{\sin x (\cos x - \sin x)} \quad (\text{Factor out } -)$$

$$= \frac{\sin^2 x}{\cos x (\sin x - \cos x)} - \frac{\cos^2 x}{\sin x (\sin x - \cos x)} \quad (\cos x - \sin x = -(\sin x - \cos x))$$

$$= \frac{\sin x \cdot \frac{\sin^2 x}{\cos x (\sin x - \cos x)}}{\sin x (\sin x - \cos x)} - \frac{\cos x \cdot \frac{\cos^2 x}{\sin x (\sin x - \cos x)}}{\sin x (\sin x - \cos x)} \quad (\text{Common denom.})$$

$$= \frac{\sin^3 x}{\cos x \sin x (\sin x - \cos x)} - \frac{\cos^3 x}{\cos x \sin x (\sin x - \cos x)} \quad (\text{Simplify})$$

$$= \frac{\sin^3 x - \cos^3 x}{\sin x \cos x (\sin x - \cos x)}$$

Add numerators

$$= \frac{(\sin x - \cos x)(\sin^2 x + \sin x \cos x + \cos^2 x)}{\sin x \cos x (\sin x - \cos x)}$$

Factor numerator

Difference of two cubes
 $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

$$= \frac{\sin^2 x + \sin x \cos x + \cos^2 x}{\sin x \cos x}$$

Simplify

$$= \frac{\sin^2 x + \cos^2 x + \sin x \cos x}{\sin x \cos x}$$

Rearrange

$$= \frac{1 + \sin x \cos x}{\sin x \cos x}$$

Pyth id.

$$= \frac{1}{\sin x \cos x} + \frac{\sin x \cos x}{\sin x \cos x}$$

Split fraction

$$= \frac{1}{\sin x} \cdot \frac{1}{\cos x} + 1$$

Simplify

$$= \csc x \sec x + 1$$

$$\therefore \frac{\tan x}{1 - \cot x} + \frac{\cot x}{1 - \tan x} = \csc x \sec x + 1.$$

$$(49) \quad \text{Pf: } \cos^3 x = \cos^2 x \cdot \cos x$$

Rewrite
 Pyth id.

$$\therefore \cos^3 x = (1 - \sin^2 x) \cos x$$

$$(51) \quad \text{Pf: } \sin^5 x = \sin^4 x \cdot \sin x$$

Rewrite

$$= (\sin^2 x)^2 \cdot \sin x$$

$\sin^4 x = \sin^2 x \cdot \sin^2 x = (\sin^2 x)^2$

$$= (1 - \cos^2 x)^2 \cdot \sin x$$

Pyth. id.

$$= (1 - \cos^2 x)(1 - \cos^2 x) \cdot \sin x$$

$$= (1 - \cos^2 x - \cos^2 x + \cos^4 x) \cdot \sin x$$

Distribute

$$= (1 - 2\cos^2 x + \cos^4 x) \sin x$$

Simplify

$$\therefore \sin^5 x = (1 - 2\cos^2 x + \cos^4 x) \sin x$$

$$\textcircled{1} \quad f(x) = \cos x \cdot \cot x \stackrel{?}{=} \sin x$$

$$\begin{aligned} \text{Pf: } \cos x \cdot \cot x &= \frac{\cos x}{1} \cdot \frac{\cos x}{\sin x} \\ &= \frac{\cos x \cdot \cos x}{\sin x} \\ &= \frac{\cos^2 x}{\sin x} \\ &\neq \sin x \end{aligned}$$

$$\therefore \cos x \cdot \cot x = \sin x$$

is NOT an identity.

$$\textcircled{9} \quad f(x) = (\sin^3 x)(1 + \cot^2 x) \stackrel{?}{=} \sin x$$

$$\begin{aligned} \text{Pf: } (\sin^3 x)(1 + \cot^2 x) &= \sin^3 x \cdot \csc^2 x \\ &= \frac{\sin^3 x}{1} \cdot \frac{1}{\sin^2 x} \\ &= \frac{\sin^3 x}{\sin^2 x} \\ &= \frac{\cancel{\sin^2 x} \cdot \sin x}{\cancel{\sin^2 x}} \\ &= \sin x \end{aligned}$$

$$\therefore (\sin^3 x)(1 + \cot^2 x) = \sin x$$

is an identity.

Graphical method: (sketch the graph)

* Graph $f(x) = \cos x \cdot \cot x$ (left hand side) and $f(x) = \sin x$ (right hand side) in the same viewing window. Notice that the graphs don't overlap for all values of x . $\therefore \cos x \cdot \cot x = \sin x$ is NOT an identity.

Graphical method: (sketch the graph)

* Graph $f(x) = (\sin^3 x)(1 + \cot^2 x)$ (L.H.S) and $f(x) = \sin x$ (R.H.S.) in the same viewing window. Notice that the graphs overlap each other for all values of x . $\therefore (\sin^3 x)(1 + \cot^2 x) = \sin x$ is an identity.

$$\textcircled{23} \quad \text{Pf: } \frac{1 + \tan^2 x}{\sin^2 x + \cos^2 x} = \frac{\sec^2 x}{1} = \sec^2 x \quad (\text{Pyth id and simplify})$$

$$\begin{aligned} \textcircled{25} \quad \text{Pf: } \frac{\cos \beta}{1 + \sin \beta} &= \frac{\cos \beta}{1 + \sin \beta} \cdot 1 \\ &= \frac{\cos \beta}{(1 + \sin \beta)} \cdot \frac{(1 - \sin \beta)}{(1 - \sin \beta)} \quad \text{Multiply by 1} \\ &= \frac{\cos \beta (1 - \sin \beta)}{1 - \cancel{\sin \beta} + \cancel{\sin \beta} - \sin^2 \beta} \quad \text{Distribute} \\ &= \frac{\cos \beta (1 - \sin \beta)}{1 - \sin^2 \beta} \quad \text{Simplify} \\ &= \frac{\cos \beta (1 - \sin \beta)}{\cos^2 \beta} \quad \text{Pyth. id} \\ &= \frac{\cos \beta (1 - \sin \beta)}{\cos \beta \cos \beta} \quad \text{Simplify} \\ &= \frac{1 - \sin \beta}{\cos \beta} \\ \therefore \frac{\cos \beta}{1 + \sin \beta} &= \frac{1 - \sin \beta}{\cos \beta}. \end{aligned}$$

(27) Pf: $\frac{\tan^2 x}{\sec x + 1} = \frac{\sec^2 x - 1}{\sec x + 1}$ (Pythag. id)

$$= \frac{(\sec x + 1)(\sec x - 1)}{\sec x + 1}$$
 (Factor)
$$= \sec x - 1$$
 (Simplify)
$$= \frac{1}{\cos x} - 1$$
 (Recip. id)
$$= \frac{1}{\cos x} - \frac{\cos x}{\cos x}$$
 (Common denom)
$$= \frac{1 - \cos x}{\cos x}$$
 (Add numerators)

$$\therefore \frac{\tan^2 x}{\sec x + 1} = \frac{1 - \cos x}{\cos x}.$$

(31) Pf: $\cos^4 x - \sin^4 x = \cos^2 x \cdot \cos^2 x - \sin^2 x \cdot \sin^2 x$ (Factor)

$$= (\cos^2 x)^2 - (\sin^2 x)^2$$
 (Difference of two squares:
 $a^2 - b^2 = (a+b)(a-b)$)
$$= \underbrace{(\cos^2 x + \sin^2 x)}_1 (\cos^2 x - \sin^2 x)$$
 (Factor)
$$= 1 \cdot (\cos^2 x - \sin^2 x)$$
 (Pyth id)
$$= \cos^2 x - \sin^2 x$$

$$\therefore \cos^4 x - \sin^4 x = \cos^2 x - \sin^2 x.$$

(43) Pf: $\cos^5 x = \cos^3 x \cdot \cos^2 x$ (Factor)

$$= \cos^3 x \cdot (1 - \sin^2 x)$$
 (Pyth. id)
$$= \cos x \cdot \cos^2 x (1 - \sin^2 x)$$
 (Factor)
$$= \cos x (1 - \sin^2 x) (1 - \sin^2 x)$$
 (Pyth id)
$$= \cos x (1 - \sin^2 x - \sin^2 x + \sin^4 x)$$
 (Distribute)
$$= \cos x (1 - 2\sin^2 x + \sin^4 x)$$

$$\therefore \cos^5 x = \cos x (1 - 2\sin^2 x + \sin^4 x).$$

(61) First step: (E)

$$\frac{\sin x}{1 - \cos x} = \frac{\sin x}{1 - \cos x} \cdot \underbrace{\frac{1 + \cos x}{1 + \cos x}}$$

Multiplying by this fancy form of 1 is efficient because after distributing $(1 - \cos x)(1 + \cos x)$ we arrive at $1 - \cos^2 x = \sin^2 x$. We need to get to $\sin x$ in denominator.

(62) Notice that:

$$\begin{aligned} \tan \theta + \sec \theta &= \frac{\sin \theta}{\cos \theta} + \frac{1}{\cos \theta} \\ &= \frac{\sin \theta + 1}{\cos \theta} \end{aligned}$$

so (C) is an intermediate expression in the proof of $\tan \theta + \sec \theta = \frac{\cos \theta}{1 - \sin \theta}$.

PAb-7 p. 425 # 3, 5, 11, 14, 19, 23, 25, 27

(3) $\sin 75^\circ = \sin(30^\circ + 45^\circ)$

$$\begin{aligned} &= \sin(30^\circ)\cos(45^\circ) + \cos(30^\circ)\sin(45^\circ) \\ &= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} \\ &= \boxed{\frac{\sqrt{2} + \sqrt{6}}{4}} \\ \therefore \sin 75^\circ &= \frac{\sqrt{2} + \sqrt{6}}{4}. \end{aligned}$$

(5) $\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$

$$\begin{aligned} &= \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) \\ &= \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right) \\ &= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} \\ &= \boxed{\frac{\sqrt{2} + \sqrt{6}}{4}} \\ \therefore \cos\left(\frac{\pi}{12}\right) &= \frac{\sqrt{2} + \sqrt{6}}{4}. \end{aligned}$$

(11) $\sin 42^\circ \cos 17^\circ - \cos 42^\circ \sin 17^\circ = \sin(42^\circ - 17^\circ) = \boxed{\sin 25^\circ}$ "sine of an angle"

(14) $\sin\frac{\pi}{3}\cos\frac{\pi}{7} - \sin\frac{\pi}{7}\cos\frac{\pi}{3} = \sin\frac{\pi}{3}\cos\frac{\pi}{7} - \cos\frac{\pi}{3}\sin\frac{\pi}{7}$ {Rearrange}

$$\begin{aligned} &= \sin\left(\frac{\pi}{3} - \frac{\pi}{7}\right) \\ &= \sin\left(\frac{\pi}{3} \cdot \frac{7}{7} - \frac{\pi}{7} \cdot \frac{3}{3}\right) \{ \text{common denominator} \} \\ &= \sin\left(\frac{7\pi}{21} - \frac{3\pi}{21}\right) \\ &= \boxed{\sin\left(\frac{4\pi}{21}\right)}$$
 "sine of an angle"

(19) $\sin(3x)\cos x - \cos(3x)\sin x = \sin(3x - x) = \boxed{\sin(2x)}$ "sine of an angle"

(23) Pt: $\sin(x - \frac{\pi}{2}) = \sin x \cos\left(\frac{\pi}{2}\right) - \cos x \sin\left(\frac{\pi}{2}\right)$

$$\begin{aligned} &= \sin x \cdot 0 - \cos x \cdot 1 \\ &= 0 - \cos x \\ &= \boxed{-\cos x} \end{aligned}$$

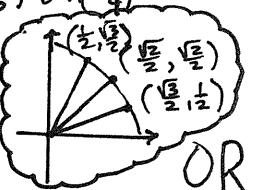
$$\therefore \sin\left(x - \frac{\pi}{2}\right) = -\cos x.$$

$$\begin{aligned}
 \textcircled{25} \text{ If: } \cos(x - \frac{\pi}{2}) &= \cos x \cdot \cos(\frac{\pi}{2}) + \sin x \cdot \sin(\frac{\pi}{2}) \\
 &= \cos x \cdot 0 + \sin x \cdot 1 \\
 &= 0 + \sin x \\
 &= \boxed{\sin x}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{27} \text{ If: } \sin(x + \frac{\pi}{6}) &= \sin x \cdot \cos(\frac{\pi}{6}) + \cos x \cdot \sin(\frac{\pi}{6}) \\
 &= \sin x \cdot \frac{\sqrt{3}}{2} + \cos x \cdot \frac{1}{2} \\
 &= \boxed{\frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x} \quad \text{or} \quad \boxed{\frac{1}{2}(\sqrt{3} \sin x + \cos x)}
 \end{aligned}$$

$$\therefore \sin(x + \frac{\pi}{6}) = \frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x.$$

$$\begin{aligned}
 \textcircled{1} \quad \tan \frac{5\pi}{12} &= \tan\left(\frac{2\pi}{12} + \frac{3\pi}{12}\right) \\
 &= \tan\left(\frac{\pi}{6} + \frac{\pi}{4}\right) \\
 &= \frac{\tan\left(\frac{\pi}{6}\right) + \tan\left(\frac{\pi}{4}\right)}{1 - \tan\left(\frac{\pi}{6}\right) \cdot \tan\left(\frac{\pi}{4}\right)} \\
 &= \frac{\frac{1}{\sqrt{3}} + \frac{\sqrt{2}}{\sqrt{2}}}{1 - \left(\frac{1}{\sqrt{3}}\right)\left(\frac{\sqrt{2}}{\sqrt{2}}\right)} \\
 &= \frac{\frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}} \cdot 1} \\
 &= \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}}} \quad \text{OR} \\
 &= \frac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}}} \quad \text{Multiply by } \frac{\sqrt{3}}{\sqrt{3}} \\
 &= \boxed{\frac{1 + \sqrt{3}}{\sqrt{3} - 1}} \quad \text{or} \quad \boxed{\frac{\sqrt{3} + 1}{\sqrt{3} - 1}}
 \end{aligned}$$



Best answer:

$$\begin{aligned}
 \frac{\sqrt{3} + 1}{\sqrt{3} - 1} \cdot \frac{(\sqrt{3} + 1)}{(\sqrt{3} + 1)} &= \frac{3 + \sqrt{3} + \sqrt{3} + 1}{3 + \sqrt{3} - \sqrt{3} - 1} \\
 &= \frac{4 + 2\sqrt{3}}{2} \\
 &= \frac{4}{2} + \frac{2\sqrt{3}}{2} \quad \text{split fraction} \\
 &= \boxed{2 + \sqrt{3}}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{9} \quad \cos\left(\frac{7\pi}{12}\right) &= \cos\left(\frac{4\pi}{12} + \frac{3\pi}{12}\right) \\
 &= \cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right) \\
 &= \cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right) \\
 &= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} \\
 &= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} \\
 &= \boxed{\frac{\sqrt{2} - \sqrt{6}}{4}}
 \end{aligned}$$

$$\therefore \cos\left(\frac{7\pi}{12}\right) = \frac{\sqrt{2} - \sqrt{6}}{4}.$$

$$\begin{aligned}
 \tan \frac{5\pi}{12} &= \tan\left(\frac{8\pi}{12} - \frac{3\pi}{12}\right) \\
 &= \frac{\sin\left(\frac{2\pi}{3} - \frac{\pi}{4}\right)}{\cos\left(\frac{2\pi}{3} - \frac{\pi}{4}\right)} \\
 &= \frac{\sin\left(\frac{2\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{2\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{2\pi}{3}\right)\cos\left(\frac{\pi}{4}\right) + \sin\left(\frac{2\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)} \\
 &= \frac{\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)}{\left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}} \\
 &= \frac{\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}}{-\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}} \\
 &= \frac{\sqrt{6} + \sqrt{2}}{-\sqrt{2} + \sqrt{6}} \quad \text{Multiply by 4} \\
 &= \frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}} \\
 &= \frac{\sqrt{2}(\sqrt{3} + 1)}{\sqrt{2}(\sqrt{3} - 1)} \quad \text{Factor } \sqrt{2} \\
 &= \boxed{\frac{\sqrt{3} + 1}{\sqrt{3} - 1}} \quad \text{or} \quad \boxed{2 + \sqrt{3}} \\
 \therefore \tan \frac{5\pi}{12} &= 2 + \sqrt{3}.
 \end{aligned}$$

$$\textcircled{15} \quad \frac{\tan 19^\circ + \tan 47^\circ}{1 - \tan 19^\circ \cdot \tan 47^\circ} = \tan(19^\circ + 47^\circ) = \boxed{\tan 66^\circ} \quad \text{"tangent of an angle"}$$

$$\begin{aligned}
 \textcircled{17} \quad \cos\left(\frac{\pi}{7}\right)\cos x + \sin\left(\frac{\pi}{7}\right)\sin x &= \boxed{\cos\left(\frac{\pi}{7} - x\right)} \\
 \text{or } \boxed{\cos(x - \frac{\pi}{7})} \quad \text{Do you see why?}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{29} \quad \text{pf: } \tan(\theta + \frac{\pi}{4}) &= \frac{\tan(\theta) + \tan(\frac{\pi}{4})}{1 - \tan(\theta)\tan(\frac{\pi}{4})} \\
 &= \frac{\tan \theta + 1}{1 - \tan \theta \cdot 1} \\
 &= \frac{1 + \tan \theta}{1 - \tan \theta} \\
 \therefore \tan(\theta + \frac{\pi}{4}) &= \frac{1 + \tan \theta}{1 - \tan \theta}
 \end{aligned}$$

$$\begin{aligned}
 \tan \frac{\pi}{4} &= \frac{\sin \frac{\pi}{4}}{\cos \frac{\pi}{4}} \\
 &= \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} \\
 &= 1
 \end{aligned}$$

(37) Pf: $\sin\left(\frac{\pi}{2} - u\right) = \sin\left(\frac{\pi}{2}\right)\cos(u) - \cos\left(\frac{\pi}{2}\right)\sin(u)$
 $= 1 \cdot \cos(u) - 0 \cdot \sin(u)$
 $= \cos(u) - 0$
 $= \cos u$
 $\therefore \sin\left(\frac{\pi}{2} - u\right) = \cos u.$

(39) Pf: $\cot\left(\frac{\pi}{2} - u\right) = \frac{\cos\left(\frac{\pi}{2} - u\right)}{\sin\left(\frac{\pi}{2} - u\right)}$ Cot $\theta = \frac{\cos\theta}{\sin\theta}$
 $= \frac{\sin u}{\cos u}$ Cofunction identity
 $= \tan u$
 $\therefore \cot\left(\frac{\pi}{2} - u\right) = \tan u.$

OR

Pf: $\cot\left(\frac{\pi}{2} - u\right) = \frac{\cos\left(\frac{\pi}{2} - u\right)}{\sin\left(\frac{\pi}{2} - u\right)}$
 $= \frac{\cos\left(\frac{\pi}{2}\right)\cos(u) + \sin\left(\frac{\pi}{2}\right)\sin(u)}{\sin\left(\frac{\pi}{2}\right)\cos(u) - \cos\left(\frac{\pi}{2}\right)\sin(u)}$
 $= \frac{0 \cdot \cos(u) + 1 \cdot \sin(u)}{1 \cdot \cos(u) - 0 \cdot \sin(u)}$
 $= \frac{0 + \sin(u)}{\cos(u) - 0}$
 $= \frac{\sin u}{\cos u}$
 $= \tan u$

$\therefore \cot\left(\frac{\pi}{2} - u\right) = \tan u.$

(41) Pf: $\csc\left(\frac{\pi}{2} - u\right) = \frac{1}{\sin\left(\frac{\pi}{2} - u\right)} \stackrel{\text{Cofunction identity}}{=} \frac{1}{\cos u} = \sec u ; \therefore \csc\left(\frac{\pi}{2} - u\right) = \sec u.$

OR

Pf: $\csc\left(\frac{\pi}{2} - u\right) = \frac{1}{\sin\left(\frac{\pi}{2} - u\right)}$
 $= \frac{1}{\sin\left(\frac{\pi}{2}\right)\cos(u) - \cos\left(\frac{\pi}{2}\right)\sin(u)}$
 $= \frac{1}{1 \cdot \cos(u) - 0 \cdot \sin(u)}$
 $= \frac{1}{\cos u - 0}$
 $= \frac{1}{\cos u}$
 $\therefore \csc\left(\frac{\pi}{2} - u\right) = \sec u. \quad = \sec u$

(43) Express $y = 3\sin x + 4\cos x$ as a sinusoid in form $y = a\sin(bx+c)$.

Notice: $a\sin(bx+c) = a(\sin(bx)\cos(c) + \cos(bx)\sin(c))$ (sum identity)

$= a\sin(bx)\cos(c) + a\cos(bx)\sin(c)$ (Distribute a inside parenthesis)

$= a\cos(c)\cdot\sin(bx) + a\sin(c)\cdot\cos(bx)$ (Rearrange)

$$3\sin x + 4\cos x = a\cos(c)\cdot\sin(bx) + a\sin(c)\cdot\cos(bx)$$

① $3 = a\cos(c)$ ② $4 = a\sin(c)$

Also, $\sin x = \sin(bx)$ and $\cos x = \cos(bx)$

$$\sin(1 \cdot x) = \sin(bx)$$

$$\therefore b=1$$

$$\therefore b=1$$

① $3 = a\cos(c)$

② $4 = a\sin(c)$

$$3^2 = a^2 \cos^2(c)$$

$$4^2 = a^2 \sin^2(c)$$

$$9 = a^2 \cos^2(c)$$

$$16 = a^2 \sin^2(c)$$

→ ADD EQUATIONS ←

$$9 = a^2 \cos^2(c)$$

$$+ 16 = a^2 \sin^2(c)$$

$$25 = a^2 \cos^2(c) + a^2 \sin^2(c)$$

$$25 = a^2 (\cos^2(c) + \sin^2(c))$$

$$25 = a^2 \cdot 1$$

$$25 = a^2$$

$$\pm 5 = a$$

choose $a = 5$ or $a = -5$; positive value always better to work with

$$a = 5 \Rightarrow 3 = 5\cos(c), 4 = 5\sin(c)$$

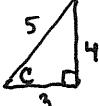
$$\frac{3}{5} = \cos(c)$$

$$\frac{4}{5} = \sin(c)$$

$$\cos^{-1}\left(\frac{3}{5}\right) = c$$

$$\sin^{-1}\left(\frac{4}{5}\right) = c$$

Same value of $c = 0.927$ radians

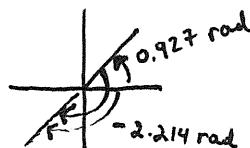


$$\therefore y = 3\sin x + 4\cos x$$

$$= 5\sin(x + 0.927)$$

$$\therefore y = 5\sin(x + 0.927)$$

This answer is just fine



$$a = -5 \Rightarrow 3 = -5\cos(c), 4 = -5\sin(c)$$

$$-\frac{3}{5} = \cos(c)$$

$$\frac{4}{5} = \sin(c)$$

$$\cos^{-1}\left(-\frac{3}{5}\right) = c$$

$$\sin^{-1}\left(\frac{4}{5}\right) = c$$

$$2.214 \text{ rad} = c$$

$$-0.927 = c$$

↳ Won't produce correct graphs ←

$$\therefore y = -5\sin(x - 2.214)$$

or

$$y = -5\sin(x - 4.369)$$

Need to watch out for phase shift

Graph to check

PA 6-9 p. 432 #1, 3, 11, 15, 19, 21

$$\begin{aligned} \textcircled{1} \text{ pf: } \cos 2u &= \cos(u+u) \\ &= \cos u \cdot \cos u - \sin u \cdot \sin u \\ &= \cos^2 u - \sin^2 u \\ \therefore \cos 2u &= \cos^2 u - \sin^2 u \end{aligned}$$

$$\begin{aligned} \textcircled{3} \text{ pf: } \cos 2u &= \underline{\cos^2 u} - \underline{\sin^2 u} \quad \text{see } \textcircled{2} \\ &= 1 - \sin^2 u - \sin^2 u \quad \text{Pyth id} \\ &= 1 - 2\sin^2 u \quad \text{Combine like terms} \end{aligned}$$

$$\begin{aligned} \textcircled{11} \quad \underline{\sin 2\theta} + \cos \theta &= \underline{2\sin \theta \cdot \cos \theta} + \cos \theta \quad \text{Double angle id. for sine} \\ &= \cos \theta (2\sin \theta + 1) \quad \text{Factor cos } \theta \end{aligned}$$

$$\begin{aligned} \textcircled{15} \quad \text{Pf: } \sin 4x &= \sin(2 \cdot 2x). \text{ Let } 2x = A. \text{ Then,} \\ \sin 4x &= \sin(2 \cdot 2x) = \sin(2A) = 2\sin A \cdot \cos A. \end{aligned}$$

By substituting $2x$ back in for A in $2\sin A \cdot \cos A$
we arrive at $2\sin 2x \cdot \cos 2x$.

$$\therefore \sin 4x = 2\sin 2x \cdot \cos 2x$$

$$\begin{aligned} \textcircled{19} \quad \text{Pf: } \sin 3x &= \sin(x+2x) \\ &= \sin x \cdot \underline{\cos 2x} + \cos x \cdot \underline{\sin 2x} \quad \text{sum id. for sine} \\ &= \sin x (2\cos^2 x - 1) + \cos x \cdot 2\sin x \cdot \cos x \quad \text{Double angle id.} \\ &= \sin x (2\cos^2 x - 1) + 2\sin x \cdot \cos^2 x \quad \text{simplify} \\ &= 2\sin x \cdot \cos^2 x - \sin x + 2\sin x \cdot \cos^2 x \quad \text{Distribute} \\ &= \sin x (2\cos^2 x - 1 + 2\cos^2 x) \quad \text{like terms} \\ &= \sin x (4\cos^2 x - 1) \\ \therefore \sin 3x &= \sin x (4\cos^2 x - 1) \end{aligned}$$

$$\begin{aligned} \textcircled{21} \quad \text{Pf: } \cos 4x &= \cos(2x+2x) \\ &= \underline{\cos 2x} \cdot \underline{\cos 2x} - \underline{\sin 2x} \cdot \underline{\sin 2x} \\ &= (1 - 2\sin^2 x)(2\cos^2 x - 1) - 2\sin x \cdot \cos x \cdot 2\sin x \cdot \cos x \quad \text{Double angle id.} \\ &= \underline{2\cos^2 x - 1} - \underline{4\sin^2 x \cdot \cos^2 x} + \underline{2\sin^2 x} = 4\sin^2 x \cdot \cos^2 x \quad \text{Distribute and multiply} \\ &= 2\cos^2 x + 2\sin^2 x - 1 - 8\sin^2 x \cdot \cos^2 x \quad \text{like terms and rearrange} \\ &= 2(\cos^2 x + \sin^2 x) - 1 - 8\sin^2 x \cdot \cos^2 x \quad \text{Factor 2} \\ &= 2(1) - 1 - 8\sin^2 x \cdot \cos^2 x \quad \text{Pyth id} \\ &= \underline{2} - \underline{1} - 8\sin^2 x \cdot \cos^2 x \quad \text{simplify} \\ &= 1 - 8\sin^2 x \cdot \cos^2 x. \end{aligned}$$

OR Shorter solution on the next page →

Pf: $\cos 4x = \cos(2 \cdot 2x)$ (Let $2x = A$)

$$= \cos(2A) \quad (\text{Substitute } A)$$

$$= 1 - 2\sin^2 A \quad (\text{Double angle id.})$$

$$= 1 - 2\sin A \cdot \sin A \quad (\text{Factor } \sin^2 A)$$

$$= 1 - 2\sin(2x) \cdot \sin(2x) \quad (\text{Substitute } 2x \text{ back})$$

$$= 1 - 2 \cdot 2\sin x \cos x \cdot 2\sin x \cos x \quad (\text{Double angle id.})$$

$$= 1 - 2 \cdot 2 \cdot 2 \cdot \sin x \cdot \sin x \cdot \cos x \cdot \cos x \quad (\text{Rearrange})$$

$$= 1 - 8\sin^2 x \cos^2 x \quad (\text{Multiply})$$

$$\therefore \cos 4x = 1 - 8\sin^2 x \cos^2 x$$

(5) Solve for x on $[0, 2\pi]$.

$$\sin 2x = 2 \sin x \quad (\text{Set } = 0)$$

$$\underline{\sin 2x} - 2 \sin x = 0 \quad (\text{Double L identity})$$

$$2 \sin x \cos x - 2 \sin x = 0 \quad (\text{Factor } 2 \sin x)$$

$$2 \sin x (\cos x - 1) = 0 \quad (\text{Set factor } = 0)$$

$$\frac{2 \sin x}{2} = 0 \quad \text{or} \quad \frac{\cos x - 1}{+1} = 0$$

$$\sin x = 0 \quad \cos x = 1$$

$$x = 0, \pi$$

$$\therefore x = 0, \pi$$

(9) $\sin 2x - \tan x = 0$

$$2 \sin x \cos x - \frac{\sin x}{\cos x} = 0$$

Quotient Identity

$$\sin x \cdot \left(2 \cos x - \frac{1}{\cos x} \right) = 0$$

Factor sinx

$$\sin x \cdot \left(\frac{2 \cos x}{1} \cdot \frac{\cos x}{\cos x} - \frac{1}{\cos x} \right) = 0$$

Common denom

$$\sin x \cdot \left(\frac{2 \cos^2 x - 1}{\cos x} \right) = 0$$

$$\sin x = 0 \quad \text{or} \quad \frac{2 \cos^2 x - 1}{\cos x} = 0$$

$$x = 0, \pi$$

$$2 \cos^2 x - 1 = 0$$

$$2 \cos^2 x = 1 \quad \sqrt{\cos^2 x} = \pm \frac{1}{2}$$

$$\cos x = \pm \frac{1}{2}$$

$$\cos x = \pm \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$\therefore x = 0, \pi, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

(11) Pf: $2 \cdot \csc 2x = 2 \cdot \frac{1}{\sin 2x}$ Recip id.

$$= \frac{2}{1} \cdot \frac{1}{2 \sin x \cos x}$$

Double angle id

$$= \frac{1}{\sin x \cdot \cos x} \cdot 1$$

$$= \frac{1}{\sin x \cdot \cos x} \cdot \frac{\sin x}{\sin x}$$

Multiply by 1

$$= \frac{1}{\sin x \cdot \sin x} \cdot \frac{\sin x}{\cos x}$$

Rearrange

$$= \frac{1}{\sin^2 x} \cdot \frac{\sin x}{\cos x}$$

Quot/Recip identities

$$= \csc^2 x \cdot \tan x$$

$$\therefore 2 \csc 2x = \csc^2 x \cdot \tan x.$$

(23) $\cos 2x + \cos x = 0$

$$2 \cos^2 x - 1 + \cos x = 0$$

$$2 \cos^2 x + \cos x - 1 = 0$$

Let $u = \cos x$. Then,

$$2u^2 + u - 1 = 0$$

$$(2u - 1)(u + 1) = 0$$

$$2u - 1 = 0 \quad \text{or} \quad u + 1 = 0$$

$$\frac{2u}{2} = \frac{1}{2}$$

$$u = \frac{1}{2}$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\therefore x = \pi, \frac{\pi}{3}, \frac{5\pi}{3}$$

(31) $\sin 15^\circ = \sin(60^\circ - 45^\circ)$

$$= \sin 60^\circ \cos 45^\circ - \cos 60^\circ \sin 45^\circ$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

$$\therefore \sin 15^\circ = \frac{\sqrt{6} - \sqrt{2}}{4} \approx 0.259 \checkmark$$

OR See solution using half-angle identity \rightarrow

$$(31) \text{ Using half-angle: } \sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}}$$

$$\sin 15^\circ = \sin\left(\frac{30^\circ}{2}\right) \quad \text{Notice } u = 30^\circ \text{ and } \frac{30^\circ}{2} = 15^\circ$$

$$= \pm \sqrt{\frac{1 - \cos 30^\circ}{2}}$$

$$= \pm \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}}$$

$$= \pm \sqrt{\frac{\frac{2}{2} - \frac{\sqrt{3}}{2}}{2}}$$

$$= \pm \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}}$$

$$= \pm \sqrt{\frac{2-\sqrt{3}}{2} \cdot \frac{1}{2}}$$

$$\rightarrow \pm \sqrt{\frac{2-\sqrt{3}}{4}}$$

$$= \pm \frac{\sqrt{2-\sqrt{3}}}{2}$$

$$= \boxed{\frac{1}{2}\sqrt{2-\sqrt{3}}}$$

$$\approx 0.259 \checkmark$$

{ positive only since $\sin 15^\circ$ is positive in QI. }

$$(35) \tan\left(\frac{7\pi}{12}\right) = \tan\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right)$$

$$= \tan\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$$

$$= \frac{\tan\left(\frac{\pi}{4}\right) + \tan\left(\frac{\pi}{3}\right)}{1 - \tan\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{3}\right)}$$

$$\tan\frac{\pi}{4} = \frac{\sin\frac{\pi}{4}}{\cos\frac{\pi}{4}} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$$

$$\tan\frac{\pi}{3} = \frac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$$

$$= \frac{1 + \sqrt{3}}{1 - 1 \cdot \sqrt{3}}$$

$$= \boxed{\frac{1+\sqrt{3}}{1-\sqrt{3}}} \quad \text{or}$$

$$= \frac{(1+\sqrt{3}) \cdot (1+\sqrt{3})}{(1-\sqrt{3}) \cdot (1+\sqrt{3})}$$

$$= \frac{1+\sqrt{3}+\sqrt{3}+3}{1+\sqrt{3}-\sqrt{3}-3}$$

$$= \frac{4+2\sqrt{3}}{-2}$$

$$= \frac{4}{-2} + \frac{2\sqrt{3}}{-2}$$

$$= \boxed{2-\sqrt{3}}$$

See next page for sol. using half-angle identity →

(35) Using half-angle identity:

$$\tan \frac{u}{2} = \begin{cases} \textcircled{1} & \pm \sqrt{\frac{1-\cos u}{1+\cos u}} \\ \textcircled{2} & \frac{1-\cos u}{\sin u} \\ \textcircled{3} & \frac{\sin u}{1+\cos u} \end{cases}$$

I will use $\tan \frac{u}{2} = \frac{1-\cos u}{\sin u}$. (Try this using (1) and (2))

$$\tan\left(\frac{7\pi}{12}\right) = \tan\left(\frac{\pi}{2} + \frac{7\pi}{6}\right)$$

$$= \tan\left(\frac{1}{2} \cdot \frac{7\pi}{6}\right)$$

$$= \tan\left(\frac{\frac{7\pi}{6}}{2}\right)$$

Do you see that $u = \frac{7\pi}{6}$?

$$= \frac{1 - \cos\left(\frac{7\pi}{6}\right)}{\sin\left(\frac{7\pi}{6}\right)}$$

$\sin \frac{7\pi}{6} = -\frac{1}{2}$

$\cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$

$$= \frac{1 - \left(-\frac{\sqrt{3}}{2}\right)}{-\frac{1}{2}}$$

$$= \frac{1 + \frac{\sqrt{3}}{2}}{-\frac{1}{2}}$$

$$= \frac{\frac{2}{2} + \frac{\sqrt{3}}{2}}{-\frac{1}{2}}$$

$$= \frac{\frac{2+\sqrt{3}}{2}}{-\frac{1}{2}}$$

$$= \frac{(2+\sqrt{3})}{2} \cdot (-\frac{2}{1})$$

$$= -\frac{2(2+\sqrt{3})}{2}$$

$$= -(2+\sqrt{3})$$

$$= \boxed{-2-\sqrt{3}} \checkmark$$

Let $u = \cos x$. Then,

$$2u^2 + u - 1 = 0$$

$$(2u-1)(u+1) = 0$$

$$2u-1=0 \text{ or } u+1=0$$

$$2u=1 \quad \downarrow \quad u=-1$$

$$u=\frac{1}{2}$$

$$\cos x = -1$$

$x = \pi$ on $[0, 2\pi]$

(43) $\cos^2 x = \sin^2\left(\frac{x}{2}\right)$

$$\cos^2 x = \left(\sin\left(\frac{x}{2}\right)\right)^2$$

$$\cos^2 x = \left(\pm \sqrt{\frac{1-\cos x}{2}}\right)^2$$

$$2 \cdot \cos^2 x = \frac{1-\cos x}{2} \cdot 2$$

$$2 \cos^2 x = 1 - \cos x$$

$$2 \cos^2 x + \cos x - 1 = 0$$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$ on $[0, 2\pi]$

General solution \rightarrow

The general solution is not restricted to the interval $[0, 2\pi]$.

The general solution is over the real numbers, i.e. on the interval $(-\infty, \infty)$.

We already know $x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ on $[0, 2\pi]$, so $x = -\frac{\pi}{3}, -\pi, -\frac{5\pi}{3}, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

is the solution on the interval $(-2\pi, 2\pi)$. A pattern emerges...

$$\left\{ \dots, -\frac{5\pi}{3}, -\frac{3\pi}{3}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{3\pi}{3}, \frac{5\pi}{3}, \dots \right\}$$

Notice numerators are odd #'s
and odd #'s are of the form
 $2n+1$, where n is an integer.

$$\therefore x = \frac{(2n+1)\pi}{3}, \text{ where } n \text{ is an integer.}$$

↳ this will generate the entire solution set.

PA 6-11 p. 439 #1, 3, 7, 11, 13, 15, 19, 25, 29

① AAS

$$\angle C = 180 - 60 - 45 = 75^\circ$$

$$\frac{\sin 45^\circ}{3.7} \times \frac{\sin 60^\circ}{a}$$

$$\frac{\sin 75^\circ}{c} \times \frac{\sin 45^\circ}{3.7}$$

$$a \cdot \frac{\sin 45^\circ}{\sin 45^\circ} = 3.7 \cdot \frac{\sin 60^\circ}{\sin 45^\circ}$$

$$\frac{3.7 \sin 75^\circ}{\sin 45^\circ} = c \cdot \frac{\sin 45^\circ}{\sin 45^\circ}$$

$$a = \frac{3.7 \cdot \sin 60^\circ}{\sin 45^\circ} = 4.532$$

$$5.054 = c$$

$$\therefore a = 4.532$$

③ AAS

$$\angle B = 180 - 100 - 35 = 45^\circ$$

$$\frac{\sin 100^\circ}{22} \times \frac{\sin 35^\circ}{c}$$

$$\frac{\sin 100^\circ}{22} \times \frac{\sin 45^\circ}{b}$$

$$c \cdot \frac{\sin 100^\circ}{\sin 100^\circ} = \frac{22 \cdot \sin 35^\circ}{\sin 100^\circ}$$

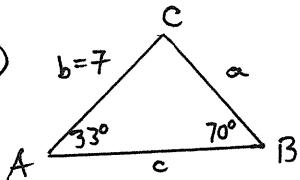
$$\frac{b \sin 100^\circ}{\sin 100^\circ} = \frac{22 \cdot \sin 45^\circ}{\sin 100^\circ}$$

$$c = 12.813$$

$$b = 15.796$$

$$⑦ A = 33^\circ, B = 70^\circ, b = 7$$

AAS



$$\angle C = 180 - 70 - 33 = 77^\circ$$

$$\frac{\sin 77^\circ}{c} \times \frac{\sin 70^\circ}{7}$$

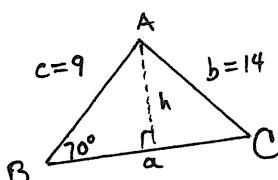
$$\frac{7 \cdot \sin 77^\circ}{\sin 70^\circ} = c \cdot \frac{\sin 70^\circ}{\sin 70^\circ}$$

$$7.258 = c$$

$$4.057 = a$$

$$⑪ B = 70^\circ, b = 14, c = 9$$

SSA



Must check to see if we have: ① no Δ, ② 1Δ ③ or 2Δs.

$$\sin 70^\circ = \frac{h}{9}$$

$$9 \cdot \sin 70^\circ = h$$

$$8.457 = h$$

height < hypotenuse

$$8.457 < 14 \quad ? \Rightarrow 1\Delta$$

$$\text{But } 9 < 14$$

$$\frac{\sin 70^\circ}{14} \times \frac{\sin C}{9}$$

$$\frac{9 \cdot \sin 70^\circ}{14} = \frac{14 \cdot \sin C}{14}$$

$$\frac{9 \cdot \sin 70^\circ}{14} = \sin C$$

$$\sin^{-1}\left(\frac{9 \cdot \sin 70^\circ}{14}\right) = \angle C$$

$$37.163^\circ = \angle C$$

store value
in calculator
 ctrl var

$$\angle A = 180 - 70 - 37.163 = 72.837^\circ$$

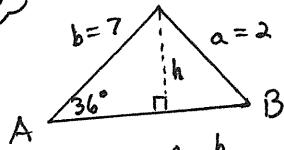
$$\frac{\sin 70^\circ}{14} \times \frac{\sin 72.837^\circ}{a}$$

$$a \cdot \frac{\sin 70^\circ}{\sin 70^\circ} = \frac{14 \cdot \sin 72.837^\circ}{\sin 70^\circ}$$

$$a = 14.235$$

$$\textcircled{13} \quad A = 36^\circ, a = 2, b = 7$$

SSA



$$\sin 36^\circ = \frac{h}{7}$$

$$7 \sin 36^\circ = h$$

$$4.114 = h$$

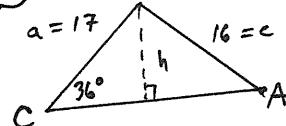
height > hypotenuse

$$4.114 > 2$$

This can't happen b/c
the hypotenuse is the
longest side in a rt. A.
 \therefore Zero

$$\textcircled{15} \quad C = 36^\circ, a = 17, c = 16$$

(SSA)



$$\sin 36^\circ = \frac{h}{17}$$

$$17 \sin 36^\circ = h$$

$$9.992 = h$$

Notice height < hypotenuse

$$9.992 < 16$$

\therefore At least 1Δ exists.

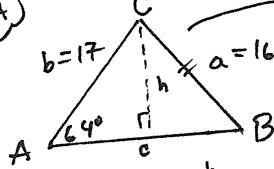
Do we have 2Δ's?

Yes, b/c $16 < 17$.

\therefore 2Δ's

$$\textcircled{19} \quad A = 64^\circ, a = 16, b = 17$$

SSA



$$\sin 64^\circ = \frac{h}{17}$$

$$17 \sin 64^\circ = h$$

$$15.279 = h$$

$15.279 < 16 \Rightarrow$ 2Δ's
AND $16 < 17$

Triangle 1

$$\frac{\sin 64^\circ}{16} \times \frac{\sin B}{17}$$

$$\frac{17 \cdot \sin 64^\circ}{16} = \frac{16 \cdot \sin B}{16}$$

$$\frac{17 \cdot \sin 64^\circ}{16} = \sin B$$

$$\sin^{-1}\left(\frac{17 \cdot \sin 64^\circ}{16}\right) = \angle B$$

$$72.74^\circ = \angle B$$

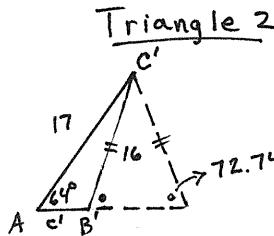
↳ store value
in calculator

$$\angle C = 180 - 64 - 72.74 = 43.26^\circ$$

$$\frac{\sin 64^\circ}{16} \times \frac{\sin 43.26^\circ}{c}$$

$$\frac{c \cdot \sin 64^\circ}{\sin 64^\circ} = \frac{16 \cdot \sin 43.26^\circ}{\sin 64^\circ}$$

$$c = 12.20$$



$$\angle A' B' C' = 180 - 72.74 = 107.26^\circ$$

$$\text{or simply } \angle B' = 107.26^\circ$$

$$\angle C' = 180 - 107.26 - 64 = 8.74^\circ$$

$$\frac{\sin 64^\circ}{16} \times \frac{\sin 8.74^\circ}{c'}$$

$$c' \cdot \frac{\sin 64^\circ}{\sin 64^\circ} = \frac{16 \cdot \sin 8.74^\circ}{\sin 64^\circ}$$

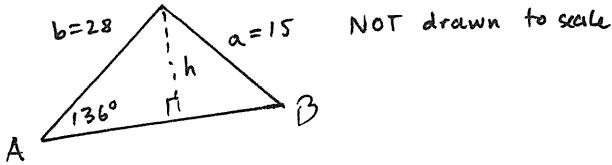
$$c' = \frac{16 \cdot \sin 8.74^\circ}{\sin 64^\circ}$$

$$c' = 2.705$$

(25) a) $\text{SAS} \therefore$ No b/c the only conditions with which the Law of Sines can be used are ASA, AAS, and SSA.

b) Again, $\text{SAS} \therefore$ No (same reason as a))

(29) $A = 136^\circ, a = 15, b = 28$



$$\sin 136^\circ = \frac{h}{28}$$

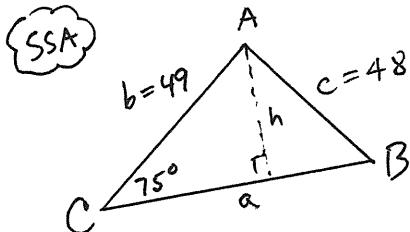
$$28 \cdot \sin 136^\circ = h$$

$$19.45 = h$$

height > hypotenuse
 $19.45 > 15$, contradiction
 \therefore No triangle is formed

Also, notice the side opposite $\angle A$ is less than 28. This cannot happen because of the converse of Pythagorean Theorem. (Across from the largest \angle must always be the longest side)

(33) $C = 75^\circ, b = 49, c = 48$



$$\sin 75^\circ = \frac{h}{49}$$

$$49 \cdot \sin 75^\circ = h$$

$$47.33 = h$$

$$47.33 < 48 \quad ? \Rightarrow 2\Delta's$$

$$\text{AND } 48 < 49$$

Triangle 1

$$\frac{\sin 75^\circ}{48} \times \frac{\sin B}{49}$$

$$\frac{49 \cdot \sin 75^\circ}{48} = \frac{48 \cdot \sin B}{48}$$

$$\frac{49 \cdot \sin 75^\circ}{48} = \sin B$$

$$\sin^{-1}\left(\frac{49 \cdot \sin 75^\circ}{48}\right) = \angle B$$

$$80.418^\circ = \angle B$$

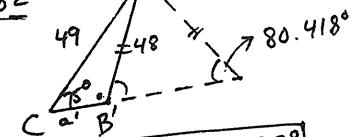
$$\angle A = 180 - 75 - 80.418 = 24.582^\circ$$

$$\frac{\sin 75^\circ}{48} \times \frac{\sin 24.582^\circ}{a}$$

$$a \cdot \frac{\sin 75^\circ}{\sin 75^\circ} = \frac{48 \cdot \sin 24.582^\circ}{\sin 75^\circ}$$

$$a = 20.672$$

Triangle 2



$$\angle B' = 180 - 80.418 = 99.582^\circ$$

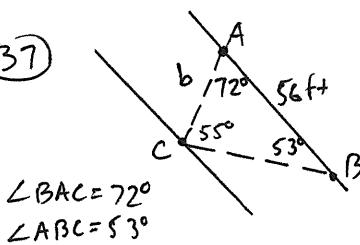
$$\angle A' = 180 - 99.582 - 75 = 5.418^\circ$$

$$\frac{\sin 5.418^\circ}{a'} \times \frac{\sin 75^\circ}{48}$$

$$\frac{48 \cdot \sin 5.418^\circ}{\sin 75^\circ} = a' \cdot \frac{\sin 75^\circ}{\sin 75^\circ}$$

$$4.692 = a'$$

(37)



$$\angle C = 180 - 53 - 72 = 55^\circ$$

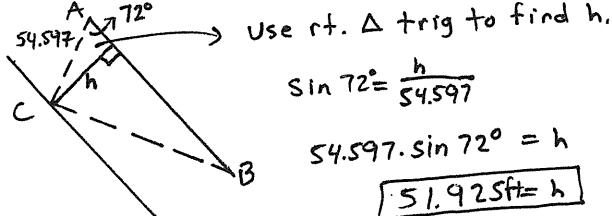
a)

$$\frac{\sin 55^\circ}{56} \times \frac{\sin 53^\circ}{b}$$

$$b \cdot \frac{\sin 55^\circ}{\sin 55^\circ} = \frac{56 \cdot \sin 53^\circ}{\sin 55^\circ}$$

$$b = 54.597 \text{ ft}$$

b) The distance between 2 canyon rims is the \perp from C to \overline{AB} , so find h.

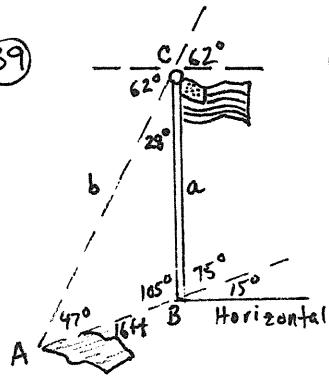


$$\sin 72^\circ = \frac{h}{54.597}$$

$$54.597 \cdot \sin 72^\circ = h$$

$$51.925 \text{ ft} = h$$

(39)



Need to find pole height.

$$\angle A = 180 - 105 - 72 = 47^\circ$$

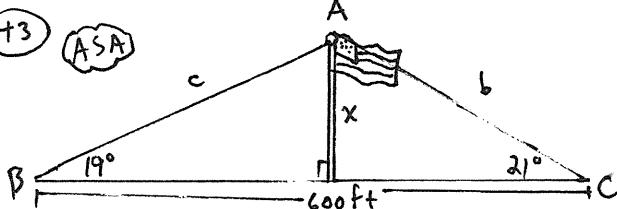
$$\frac{\sin 47^\circ}{a} \times \frac{\sin 28^\circ}{16}$$

$$\frac{16 \sin 47^\circ}{\sin 28^\circ} = a \cdot \frac{\sin 28^\circ}{\sin 28^\circ}$$

$$24.925 = a$$

\therefore The pole is 24.925 ft high.

(43)



$$180 - 19 - 21 = 140^\circ$$

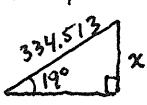
Find b or c.
I'll find little c.

$$\frac{\sin 140^\circ}{600} \times \frac{\sin 21^\circ}{c}$$

$$c \cdot \frac{\sin 140^\circ}{\sin 140^\circ} = 600 \cdot \sin 21^\circ$$

$$c = 334.513$$

Now, find x



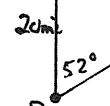
$$x = 108.907$$

\therefore The flag pole is 108.907 ft

(45)

S (ship)

$$\angle A = 180 - 52 - 33 = 95^\circ$$



$$\frac{\sin 95^\circ}{a} \times \frac{\sin 33^\circ}{20}$$

$$\frac{20 \cdot \sin 95^\circ}{\sin 33^\circ} = a \cdot \frac{\sin 33^\circ}{\sin 33^\circ}$$

$$36.582 = a$$

\therefore The ship is 36.582 miles from light house B.

$$\frac{\sin 52^\circ}{b} \times \frac{\sin 33^\circ}{20}$$

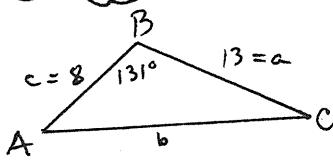
$$\frac{20 \cdot \sin 52^\circ}{\sin 33^\circ} = b \cdot \frac{\sin 33^\circ}{\sin 33^\circ}$$

$$28.937 = b$$

\therefore the ship is 28.937 miles from light house A

PA 6-13 p. 448 # 1, 3, 5, 9-17 odd

① (SAS) ∵ Use Law of Cosines



$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$b^2 = 13^2 + 8^2 - 2(13)(8)\cos 131^\circ$$

$$b = \sqrt{13^2 + 8^2 - 2(13)(8)\cos 131^\circ}$$

$$b = 19.221$$

Now, find $\angle A$ or $\angle C$.

I'll find $\angle C$.

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$8^2 = 13^2 + (19.221)^2 - 2(13)(19.221)\cos C$$

$$\frac{8^2 - 13^2 - (19.221)^2}{-2(13)(19.221)} = \frac{-2(13)(19.221)\cos C}{-2(13)(19.221)}$$

$$\frac{8^2 - 13^2 - (19.221)^2}{-2(13)(19.221)} = \cos C \quad (\text{Take inverse})$$

$$\cos^{-1}\left(\frac{8^2 - 13^2 - (19.221)^2}{-2(13)(19.221)}\right) = \angle C$$

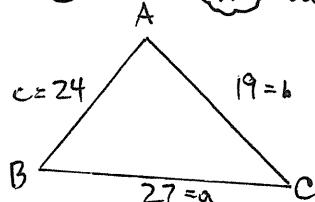
$$18.307^\circ = \angle C$$

$$\boxed{\angle A = 180 - 131 - 18.307 = 30.693^\circ}$$

③

(SSS)

Use Law of Cosines. Find $\angle A$, $\angle B$, or $\angle C$.



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$27^2 = 19^2 + 24^2 - 2(19)(24)\cos A$$

$$\frac{27^2 - 19^2 - 24^2}{-2(19)(24)} = \frac{-2(19)(24)\cos A}{-2(19)(24)}$$

$$\frac{27^2 - 19^2 - 24^2}{-2(19)(24)} = \cos A \quad (\text{Take inverse})$$

$$\cos^{-1}\left(\frac{27^2 - 19^2 - 24^2}{-2(19)(24)}\right) = \angle A$$

$$76.817^\circ = \angle A$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$24^2 = 27^2 + 19^2 - 2(27)(19)\cos C$$

$$\frac{24^2 - 27^2 - 19^2}{-2(27)(19)} = \frac{-2(27)(19)\cos C}{-2(27)(19)}$$

$$\frac{24^2 - 27^2 - 19^2}{-2(27)(19)} = \cos C \quad (\text{Take inverse})$$

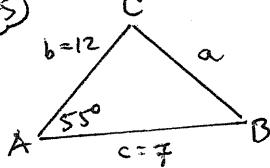
$$\cos^{-1}\left(\frac{24^2 - 27^2 - 19^2}{-2(27)(19)}\right) = \angle C$$

$$59.935^\circ = \angle C$$

$$\boxed{\therefore \angle B = 180 - 76.817 - 59.935 = 43.248^\circ}$$

⑤ $A = 55^\circ$, $b = 12$, $c = 7$

(SAS)



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = 12^2 + 7^2 - 2(12)(7)\cos 55^\circ$$

$$a = \sqrt{12^2 + 7^2 - 2(12)(7)\cos 55^\circ}$$

$$a = 9.831$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$12^2 = (9.831)^2 + 7^2 - 2(9.831)(7)\cos B$$

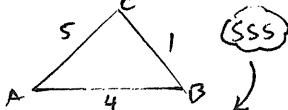
$$12^2 - (9.831)^2 - 7^2 = -2(9.831)(7)\cos B$$

$$\frac{12^2 - (9.831)^2 - 7^2}{-2(9.831)(7)} = \cos B$$

$$\cos^{-1}\left(\frac{12^2 - (9.831)^2 - 7^2}{-2(9.831)(7)}\right) = \angle B$$

$$89.314^\circ = \angle B$$

⑨ $a = 1$, $b = 5$, $c = 4$

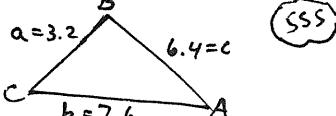


By the Δ inequality Thm,
the sum of 2 sides of a
 Δ must be greater than
the 3rd side.

Note that $4+1 < 5$.

\therefore No Δ possible

⑪ (SSS)



$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$3.2^2 = 7.6^2 + 6.4^2 - 2(7.6)(6.4)\cos A$$

$$\frac{3.2^2 - 7.6^2 - 6.4^2}{-2(7.6)(6.4)} = \cos A$$

$$\cos^{-1}\left(\frac{3.2^2 - 7.6^2 - 6.4^2}{-2(7.6)(6.4)}\right) = \angle A$$

$$24.558^\circ = \angle A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$7.6^2 = 3.2^2 + 6.4^2 - 2(3.2)(6.4)\cos B$$

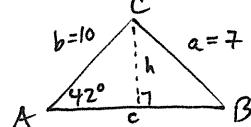
$$\frac{7.6^2 - 3.2^2 - 6.4^2}{-2(3.2)(6.4)} = \cos B$$

$$\cos^{-1}\left(\frac{7.6^2 - 3.2^2 - 6.4^2}{-2(3.2)(6.4)}\right) = \angle B$$

$$99.216^\circ = \angle B$$

$$\boxed{\therefore \angle C = 180 - 24.558 - 99.216 = 56.226^\circ}$$

$$(13) A = 42^\circ, a = 7, b = 10$$



(SSA)
Law of Sines
(can solve using Law of Cosines)

$$\sin 42^\circ = \frac{h}{10}$$

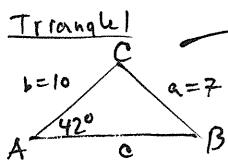
$$10 \cdot \sin 42^\circ = h$$

$$6.691 = h$$

6.691 (height) < 7 (hypotenuse)

AND $7 < 10$

\therefore , 2 possible Δ 's!



$$\frac{\sin 42^\circ}{7} \neq \frac{\sin B}{10}$$

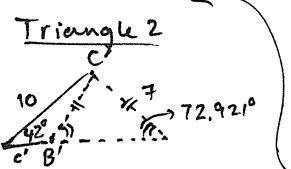
$$10 \cdot \sin 42^\circ = 7 \cdot \sin B$$

$$10 \cdot \sin 42^\circ = \sin B$$

$$\sin^{-1}\left(\frac{10 \cdot \sin 42^\circ}{7}\right) = \angle B$$

$$72.921^\circ = \angle B$$

$$\angle C = 180 - 72.921 - 42 = 65.079^\circ$$



$$\angle B' = 180 - 72.921 = 107.079^\circ$$

$$\angle C' = 180 - 42 - 107.079 = 30.921^\circ$$

$$\frac{\sin 30.921^\circ}{c'} = \frac{\sin 42^\circ}{7} \Rightarrow c' = 5.376$$

$$\frac{\sin 65.079^\circ}{c} \neq \frac{\sin 42^\circ}{7}$$

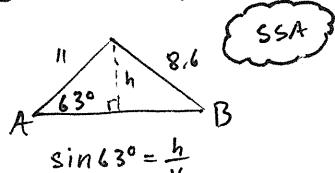
$$\frac{7 \sin 65.079^\circ}{\sin 42^\circ} = \frac{c \sin 42^\circ}{\sin 65.079^\circ}$$

$$\frac{7 \sin 65.079^\circ}{\sin 42^\circ} = c$$

$$9.487 = c$$

Show work

$$(15) \angle A = 63^\circ, a = 8.6, b = 11.1$$



SSA

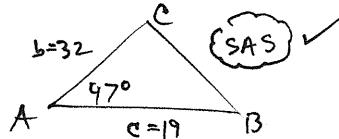
$$\sin 63^\circ = \frac{h}{11}$$

$$11 \cdot \sin 63^\circ = h$$

$$9.801 = h$$

Notice $9.801 > 8.6$; i.e.,
the height cannot be
greater than hypotenuse
of a right Δ .
 \therefore No Δ exists

$$(17) \text{Area of } \Delta = \frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B = \frac{1}{2}ab \sin C$$



SAS

$$\text{Area of } \Delta ABC = \frac{1}{2}(32)(19)\sin 47^\circ = 222.332 \text{ ft}^2$$

PA 6-14 p. 448 #21, 35-41 odd

(21) $a=4, b=5, c=8$ (SSS)

Recall the sum of 2 sides
must be greater than 3rd side

Is $a+b > c$? $4+5 > 8$, yes!

Is $b+c > a$? $5+8 > 4$, yes!

Is $a+c > b$? $4+8 > 5$, yes!

\therefore , A Δ can be formed

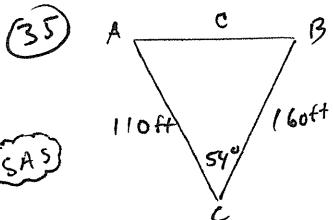
Heron's formula: Area of $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$, where
 $s = \frac{a+b+c}{2}$ (semi-perimeter)

$$s = \frac{4+5+8}{2} = \frac{17}{2}$$

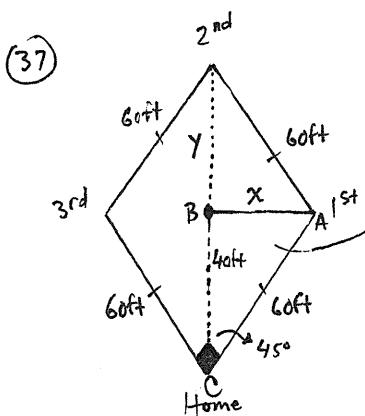
$$\therefore \text{Area of } \Delta ABC = \sqrt{\frac{17}{2}(\frac{17}{2}-4)(\frac{17}{2}-5)(\frac{17}{2}-8)}$$

$$= \sqrt{\frac{17}{2}(\frac{17}{2}-\frac{8}{2})(\frac{17}{2}-\frac{10}{2})(\frac{17}{2}-\frac{16}{2})}$$

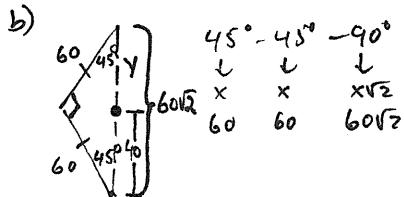
$$= \sqrt{\frac{17}{2}(\frac{9}{2})(\frac{7}{2})(\frac{1}{2})} = 8.182 \text{ units}^2$$



$$\begin{aligned}c^2 &= a^2 + b^2 - 2ab \cos C \\c^2 &= 160^2 + 110^2 - 2(160)(110) \cos 54^\circ \\c &= \sqrt{160^2 + 110^2 - 2(160)(110) \cos 54^\circ} \\c &= 130.422 \text{ ft}\end{aligned}$$

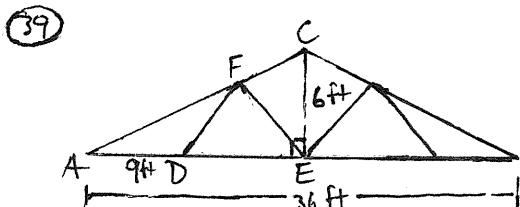


(SAS) a) $x = \sqrt{40^2 + 60^2 - 2(40)(60) \cos 45^\circ} \approx 42.5 \text{ ft}$



$$\therefore y = 60\sqrt{2} - 40 = 44.9 \text{ ft}$$

c) $B \xrightarrow{44.9=c} A$ b) $b^2 = a^2 + c^2 - 2ac \cos B$
 $a = 40$ $60^2 = 40^2 + 44.9^2 - 2(40)(44.9) \cos B$ Show work!
 $60 = b$ $\angle B = \cos^{-1} \left(\frac{60^2 - 40^2 - 44.9^2}{-2(40)(44.9)} \right) \approx 93.3^\circ$

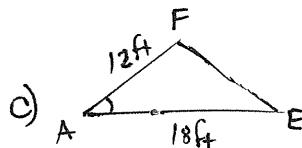


a) $\angle CAE = ?$

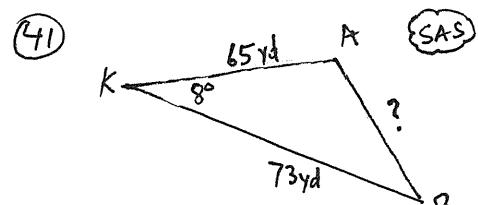
$$\begin{aligned}\tan \angle CAE &= \frac{6}{18} \\ \tan \angle A &= \frac{1}{3} \\ \angle A &= \tan^{-1} \left(\frac{1}{3} \right) \\ \therefore \angle A &= 18.435^\circ\end{aligned}$$

b) $AF = 12 \text{ ft}$. Find DF .

$$\begin{aligned}DF &= \sqrt{9^2 + 12^2 - 2(9)(12) \cos 18.435^\circ} \\ DF &= 4.482 \text{ ft}\end{aligned}$$



$$\begin{aligned}FE &= \sqrt{12^2 + 18^2 - 2(12)(18) \cos 18.435^\circ} \\ FE &= 7.629 \text{ ft}\end{aligned}$$



$$\begin{aligned}AB &= \sqrt{65^2 + 73^2 - 2(65)(73) \cos 8^\circ} \\ AB &= 12.504 \text{ yds}\end{aligned}$$