Advanced Algebra

Unit 6 Practice:

Operations & Composition of Polynomial Functions

$$f(x) \pm h(x),\ g(f(x)),\ \frac{f(x)}{g(x)}$$

Check for Understanding

Concept Check

1. **OPEN ENDED** Write an example that illustrates a property of powers. Then use multiplication or division to explain why it is true. **See margin.**

2. Determine whether $x^y \cdot x^z = x^{yz}$ is *sometimes*, *always*, or *never* true. Explain.

2. Sometimes; in general $x^y \cdot x^z = x^{y+z}$, so $x^y \cdot x^z = x^{yz}$ when $y + z = yz$, such as when $y = 2$ and $z = 2$.

3. **FIND THE ERROR** Alejandra and Kyle both simplified $\frac{2a^2b}{(-2ab^3)^{-2}}$.

Alejandra

$$\frac{2a^2b}{(-2ab^3)^{-2}} = (2a^2b)(-2ab^3)^2$$
$$= (2a^2b)(-2)^2a^2(b^3)^2$$
$$= (2a^2b)2^2a^2b^6$$
$$= 8a^4b^7$$

Kyle

$$\frac{2a^2b}{(-2ab^3)^{-2}} = \frac{2a^2b}{(-2)^{-2}a^{-2}(b^3)^{-2}}$$
$$= \frac{2a^2b}{4ab^{-6}}$$
$$= \frac{2a^2bb^6}{4a}$$
$$= \frac{ab^7}{2}$$

Who is correct? Explain your reasoning.

3. Alejandra; when Kyle used the Power of a Product property in his first step, he forgot to put an exponent of -2 on a. Also, in his second step, $(-2)^{-2}$ should be $\frac{1}{4}$, not 4.

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–9	1–3
10–12	4
13, 14	5
15	6
16, 17	7

Simplify. Assume that no variable equals 0.

4. $x^2 \cdot x^8$ **x^{10}**
5. $(2b)^4$ **$16b^4$**
6. $(n^3)^3(n^{-3})^3$ **1**
7. $\frac{30y^4}{-5y^2}$ **$-6y^2$**
8. $\frac{-2a^3b^6}{18a^2b^2}$ **$-\frac{ab^4}{9}$**
9. $\frac{81p^6q^5}{(3p^2q)^2}$ **$9p^2q^3$**
10. $\left(\frac{1}{w^4z^2}\right)^3$ **$\frac{1}{w^{12}z^6}$**
11. $\left(\frac{cd}{3}\right)^{-2}$ **$\frac{9}{c^2d^2}$**
12. $\left(\frac{-6x^6}{3x^3}\right)^{-2}$ **$\frac{1}{4x^6}$**

Express each number in scientific notation.

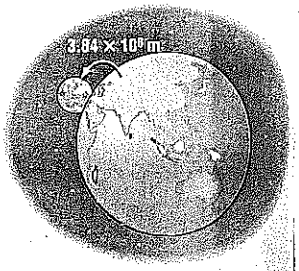
13. 421,000 **4.21×10^5**
14. 0.000862 **8.62×10^{-4}**

Evaluate. Express the result in scientific notation.

15. $(3.42 \times 10^8)(1.1 \times 10^{-5})$ **3.762×10^3**
16. $\frac{8 \times 10^{-1}}{16 \times 10^{-2}}$ **5×10^0**

Application

17. **ASTRONOMY** Refer to Example 7 on page 225. The average distance from Earth to the Moon is about 3.84×10^8 meters. How long would it take a radio signal traveling at the speed of light to cover that distance? **about 1.28 s**



Answer

29. $-24r^7s^5$

★ indicates increased difficulty

Practice and Apply

Simplify. Assume that no variable equals 0.

18. $a^2 \cdot a^6$ **a^8**
19. $b^{-3} \cdot b^7$ **b^4**
20. $(n^4)^4$ **n^{16}**
21. $(z^2)^5$ **z^{10}**
22. $(2x)^4$ **$16x^4$**
23. $(-2c)^3$ **$-8c^3$**
24. $\frac{a^2n^6}{an^5}$ **an**
25. $\frac{-y^5z^7}{y^2z^5}$ **$-y^3z^2$**
26. $(7x^3y^{-5})(4xy^3)$ **$\frac{28x^4}{y^2}$**
27. $(-3b^3c)(7b^2c^2)$ **$-21b^5c^3$**
28. $(a^3b^3)(ab)^{-2}$ **ab**
29. $(-2r^2s)^3(3rs^2)$ **$-24r^7s^5$**
30. $2x^2(6y^3)(2x^2y)$ **$24x^4y^4$**
31. $3a(5a^2b)(6ab^3)$ **$90a^4b^4$**
32. $\frac{-5x^3y^3z^4}{20x^3y^7z^4}$ **$-\frac{1}{4y^4}$**

Homework Help

For Exercises	See Examples
18–35, 60	1–3
36–39	4
40–43	1, 2
44–49, 56, 57	5
50–55, 58, 59	6, 7

Extra Practice See page 836.

33. $\frac{3a^5b^3c^3}{9a^3b^7c}$ $\frac{a^2c^2}{3b^4}$

34. $\frac{2c^3d(3c^2d^5)}{30c^4d^2}$ $\frac{cd^4}{5}$

35. $\frac{-12m^4n^8(m^3n^2)}{36m^3n}$ $-\frac{m^4n^9}{3}$

36. $\left(\frac{8a^3b^2}{16a^2b^3}\right)^4$ $\frac{a^4}{16b^4}$

37. $\left(\frac{6x^2y^4}{3x^4y^3}\right)^3$ $\frac{8y^3}{x^6}$

38. $\left(\frac{x}{y^{-1}}\right)^{-2}$ $\frac{1}{x^2y^2}$

39. $\left(\frac{v}{w^{-2}}\right)^{-3}$ $\frac{1}{v^3w^6}$

★ 40. $\frac{30a^{-2}b^{-6}}{60a^{-6}b^{-8}}$ $\frac{a^4b^2}{2}$

★ 41. $\frac{12x^{-3}y^{-2}z^{-8}}{30x^{-6}y^{-4}z^{-1}}$ $\frac{2x^3y^2}{5z^7}$

★ 42. If $2^{r+5} = 2^{2r-1}$, what is the value of r? **6**

★ 43. What value of r makes $y^{28} = y^{3r} \cdot y^7$ true? **7**

Express each number in scientific notation.

44. 462.3 **4.623×10^2**

45. 43,200 **4.32×10^4**

46. 0.0001843 **1.843×10^{-4}**

47. 0.006810 **6.81×10^{-3}**

48. 502,020,000 **5.0202×10^8**

49. 675,400,000 **6.754×10^8**

Evaluate. Express the result in scientific notation.

50. $(4.15 \times 10^3)(3.0 \times 10^6)$ **1.245×10^{10}**

51. $(3.01 \times 10^{-2})(2 \times 10^{-3})$ **6.02×10^{-5}**

52. $\frac{6.3 \times 10^5}{1.4 \times 10^3}$ **4.5×10^2**

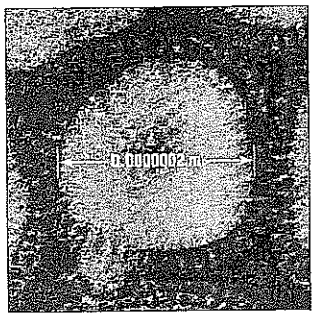
53. $\frac{9.3 \times 10^7}{1.5 \times 10^{-3}}$ **6.2×10^{10}**

54. $(6.5 \times 10^4)^2$ **4.225×10^9**

55. $(4.1 \times 10^{-4})^2$ **1.681×10^{-7}**

56. **POPULATION** The population of Earth is about 6,080,000,000. Write this number in scientific notation. **6.08×10^9**

57. **BIOLOGY** Use the diagram at the right to write the diameter of a typical flu virus in scientific notation. **2×10^{-7} m**



58. **CHEMISTRY** One gram of water contains about 3.34×10^{22} molecules. About how many molecules are in 500 grams of water? **1.67×10^{25}**

59. **RESEARCH** Use the Internet or other source to find the masses of Earth and the Sun. About how many times as large as Earth is the Sun? **about 330,000 times**

60. **CRITICAL THINKING** Determine which is greater, 100^{10} or 10^{100}. Explain. **$100^{10} = (10^2)^{10}$ or 10^{20}, and $10^{100} > 10^{20}$, so $10^{100} > 100^{10}$.**

CRITICAL THINKING For Exercises 61 and 62, use the following proof of the Power of a Power Property. **61. Definition of an exponent**

$$\begin{aligned} a^m a^n &= \overbrace{a \cdot a \cdot \ldots \cdot a}^{m \text{ factors}} \cdot \overbrace{a \cdot a \cdot \ldots \cdot a}^{n \text{ factors}} \\ &= \overbrace{a \cdot a \cdot \ldots \cdot a}^{m+n \text{ factors}} \\ &= a^{m+n} \end{aligned}$$

61. What definition or property allows you to make each step of the proof?

62. Prove the Power of a Product Property, $(ab)^m = a^m b^m$. **See margin.**

WebQuest

A scatter plot of populations will help you make a model for the data. Visit www.algebra2.com/webquest to continue work on your WebQuest project.

63. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson. **See margin.**

Why is scientific notation useful in economics?

Include the following in your answer:

- the 2000 national debt of $5,674,200,000,000 and the U.S. population of 281,000,000, both written in words and in scientific notation, and
- an explanation of how to find the amount of debt per person, with the result written in scientific notation and in standard notation.

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
4–7	1, 2
8, 9	3
10–14	4
15, 16	5

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. Justify each step. **4–5. See margin. See Students' work for justification.**

4. $f(x) = 3x + 4$
   $g(x) = 5 + x$

5. $f(x) = x^2 + 3$
   $g(x) = x - 4$

For each set of ordered pairs, find $f \circ g$ and $g \circ f$, if they exist.

6. $f = \{(-1, 9), (4, 7)\}$
   $g = \{(-5, 4), (7, 12), (4, -1)\}$
   $\{(-5, 7), (4, 9)\}$; $\{(4, 12)\}$

7. $f = \{(0, -7), (1, 2), (2, -1)\}$
   $g = \{(-1, 10), (2, 0)\}$
   $\{(2, -7)\}$; $\{(1, 0), (2, 10)\}$

Find $[g \circ h](x)$ and $[h \circ g](x)$. Justify each step.

8. $g(x) = 2x$
   $h(x) = 3x - 4$ **$6x - 8$; $6x - 4$**

9. $g(x) = x + 5$ **$x^2 + 11$; $x^2 + 10x + 31$**
   $h(x) = x^2 + 6$

If $f(x) = 3x$, $g(x) = x + 7$, and $h(x) = x^2$, find each value.

10. $f[g(3)]$ **30**

11. $g[h(-2)]$ **11**

12. $h[h(1)]$ **1**

Application

SHOPPING For Exercises 13–16, use the following information.
Mai-Lin is shopping for computer software. She finds a CD-ROM program that costs $49.99, but is on sale at a 25% discount. She also has a $5 coupon for the product.

13. Express the price of the CD after the discount and the price of the CD after the coupon using function notation. Let x represent the price of the CD, $p(x)$ represent the price after the 25% discount, and $c(x)$ represent the price after the coupon. **$p(x) = \frac{3}{4}x$; $c(x) = x - 5$**

14. Find $c[p(x)]$ and explain what this value represents.

15. Find $p[c(x)]$ and explain what this value represents.

16. Which method results in the lower sale price? Explain your reasoning. **See margin.**

14. **$32.49; price of CD when 25% discount is taken and then the coupon is subtracted**

15. **$33.74; price of CD when coupon is subtracted and then 25% discount is taken**

Practice and Apply

Homework Help	
For Exercises	See Examples
17–22	1, 2
23–28	3
29–46	4
47–55	5

Extra Practice See page 844.

Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. Justify each step. **17–22. See margin. See students' work for justification.**

17. $f(x) = x + 9$
    $g(x) = x - 9$

18. $f(x) = 2x - 3$
    $g(x) = 4x + 9$

19. $f(x) = 2x^2$
    $g(x) = 8 - x$

20. $f(x) = x^2 + 6x + 9$
    $g(x) = 2x + 6$

21. $f(x) = x^2 - 1$
    $g(x) = \frac{x}{x + 1}$

22. $f(x) = x^2 - x - 6$
    $g(x) = \frac{x - 3}{x + 2}$

For each set of ordered pairs, find $f \circ g$ and $g \circ f$ if they exist.

23. $f = \{(1, 1), (0, -3)\}$
    $g = \{(1, 0), (-3, 1), (2, 1)\}$

24. $f = \{(1, 2), (3, 4), (5, 4)\}$
    $g = \{(2, 5), (4, 3)\}$

25. $f = \{(3, 8), (4, 0), (6, 3), (7, -1)\}$
    $g = \{(0, 4), (8, 6), (3, 6), (-1, 8)\}$

26. $f = \{(4, 5), (6, 5), (8, 12), (10, 12)\}$
    $g = \{4, 6), (2, 4), (6, 8), (8, 10)\}$

27. $f = \{(2, 5), (3, 9), (-4, 1)\}$
    $g = \{(5, -4), (8, 3), (2, -2)\}$
    $\{(5, 1), (8, 9)\}$; $\{(2, -4)\}$

28. $f = \{(7, 0), (-5, 3), (8, 3), (-9, 2)\}$
    $g = \{(2, -5), (1, 0), (2, -9), (3, 6)\}$
    $\{(2, 3), (2, 2)\}$; $\{(-5, 6), (8, 6), (-9, -5)\}$

23. **$\{(1, -3), (-3, 1), (2, 1)\}$; $\{(1, 0), (0, 1)\}$**

24. **$\{(2, 4), (4, 4)\}$; $\{(1, 5), (3, 3), (5, 3)\}$**

25. **$\{(0, 0), (8, 3), (3, 3)\}$; $\{(3, 6), (4, 4), (6, 6), (7, 8)\}$**

26. **$\{(4, 5), (2, 5), (6, 12), (8, 12)\}$; does not exist**

Answers

4. $4x + 9$; $2x - 1$; $3x^2 + 19x + 2$
   $\frac{3x + 4}{x + 5}$, $x \neq -5$

5. $x^2 + x - 1$; $x^2 - x + 7$;
   $x^3 - 4x^2 + 3x - 12$; $\frac{x^2 + 3}{x - 4}$, x

16. **Discount first, then coupon; sample answer: 25% of 49.99 is greater than 25% of 44.99.**

17. $2x$; 18; $x^2 - 81$; $\frac{x + 9}{x - 9}$, $x \neq 9$

18. $6x + 6$; $-2x - 12$; $8x^2 + 6x -$
    $\frac{2x - 3}{4x + 9}$, $x \neq -\frac{9}{4}$

19. $2x^2 - x + 8$; $2x^2 + x - 8$;
    $-2x^3 + 16x^2$; $\frac{2x^2}{8 - x}$, $x \neq 8$

20. $x^2 + 8x + 15$; $x^2 + 4x + 3$;
    $2x^3 + 18x^2 + 54x + 54$; $\frac{x + 3}{2}$
    $x \neq -3$

21. $\frac{x^3 + x^2 - 1}{x + 1}$, $x \neq -1$;
    $\frac{x^3 + x^2 - 2x - 1}{x + 1}$, $x \neq -1$; $x^2 -$
    $x \neq -1$; $\frac{x^3 + x^2 - x - 1}{x}$, $x \neq 1$

22. $\frac{x^3 + x^2 - 7x - 15}{x + 2}$, $x \neq -2$;
    $\frac{x^3 + x^2 - 9x - 9}{x + 2}$, $x \neq -2$;
    $x^2 - 6x + 9$, $x \neq -2$;
    $x^2 + 4x + 4$, $x \neq -2, 3$

Check for Understanding

Concept Check

1. **OPEN ENDED** Write a quotient of two polynomials such that the remainder is 5. **Sample answer: $(x^2 + x + 5) \div (x + 1)$**

2. Explain why synthetic division cannot be used to simplify $\frac{x^3 - 3x + 1}{x^2 + 1}$.

2. The divisor contains an x^2 term.

3. **FIND THE ERROR** Shelly and Jorge are dividing $x^3 - 2x^2 + x - 3$ by $x - 4$.

Shelly

4	1	-2	1	-3
		4	-24	100
	1	-6	25	-103

Jorge

4	1	-2	1	-3
		4	8	36
	1	2	9	33

Who is correct? Explain your reasoning.

3. Jorge; Shelly is subtracting in the columns instead of adding.

10. $x^2 + 11x - 34 + \frac{60}{x + 2}$

11. $b^3 + b - 1$

Guided Practice

Simplify. **7.** $3a^3 - 9a^2 + 7a - 6$ **8.** $z^4 + 2z^3 + 4z^2 + 5z + 10$

GUIDED PRACTICE KEY	
Exercises	Examples
4, 5	1
6–10	2, 4
11, 14	3
12,13	5

4. $\frac{6xy^2 - 3xy + 2x^2y}{xy}$ **$6y - 3 + 2x$**

5. $(5ab^2 - 4ab + 7a^2b)(ab)^{-1}$ **$5b - 4 + 7a$**

6. $(x^2 - 10x - 24) \div (x + 2)$ **$x - 12$**

7. $(3a^4 - 6a^3 - 2a^2 + a - 6) \div (a + 1)$

8. $(z^5 - 3z^2 - 20) \div (z - 2)$

9. $(x^3 + y^3) \div (x + y)$ **$x^2 - xy + y^2$**

10. $\frac{x^3 + 13x^2 - 12x - 8}{x + 2}$

11. $(b^4 - 2b^3 + b^2 - 3b + 2)(b - 2)^{-1}$

12. $(12y^2 + 36y + 15) \div (6y + 3)$ **$2y + 5$**

13. $\frac{9b^2 + 9b - 10}{3b - 2}$ **$3b + 5$**

Standardized Test Practice

14. Which expression is equal to $(x^2 - 4x + 6)(x - 3)^{-1}$? **B**

(A) $x - 1$

(B) $x - 1 + \frac{3}{x - 3}$

(C) $x - 1 - \frac{3}{x - 3}$

(D) $-x + 1 - \frac{3}{x - 3}$

★ indicates increased difficulty

Practice and Apply

Simplify.

15. $\frac{9a^3b^2 - 18a^2b^3}{3a^2b}$ **$3ab - 6b^2$**

16. $\frac{5xy^2 - 6y^3 + 3x^2y^3}{xy}$ **$5y - \frac{6y^2}{x} + 3xy^2$**

17. $2c^2 - 3d + 4d^2$

17. $(28c^3d - 42cd^2 + 56cd^3) \div (14cd)$

18. $4n^2 + 3mn - 5m$

18. $(12mn^3 + 9m^2n^2 - 15m^2n) \div (3mn)$

19. $(2y^3z + 4y^2z^2 - 8y^4z^5)(yz)^{-1}$ **$2y^2 + 4yz - 8y^3z^4$**

20. $(a^3b^2 - a^2b + 2a)(-ab)^{-1}$ **$-a^2b + a - \frac{2}{b}$**

Homework Help

For Exercises	See Examples
15–20, 51	1
21–34, 49, 50, 52–54	2, 4
35–38	3, 4
39–48	2, 3, 5

Extra Practice See page 1041.

21–48. See pp. 283A–283B.

21. $(b^3 + 8b^2 - 20b) \div (b - 2)$

22. $(x^2 - 12x - 45) \div (x + 3)$

23. $(n^3 + 2n^2 - 5n + 12) \div (n + 4)$

24. $(2c^3 - 3c^2 + 3c - 4) \div (c - 2)$

25. $(x^4 - 3x^3 + x^2 - 5) \div (x + 2)$

26. $(6w^5 - 18w^2 - 120) \div (w - 2)$

27. $(x^3 - 4x^2) \div (x - 4)$

28. $(x^3 - 27) \div (x - 3)$

29. $\dfrac{y^3 + 3y^2 - 5y - 4}{y + 4}$

30. $\dfrac{m^3 + 3m^2 - 7m - 21}{m + 3}$

31. $\dfrac{a^4 - 5a^3 - 13a^2 + 10}{a + 1}$

32. $\dfrac{2m^4 - 5m^3 - 10m + 8}{m - 3}$

33. $\dfrac{x^5 - 7x^3 + x + 1}{x + 3}$

34. $\dfrac{3c^5 + 5c^4 + c + 5}{c + 2}$

35. $(g^2 + 8g + 15)(g + 3)^{-1}$

36. $(2b^3 + b^2 - 2b + 3)(b + 1)^{-1}$

37. $(t^5 - 3t^2 - 20)(t - 2)^{-1}$

38. $(y^5 + 32)(y + 2)^{-1}$

39. $(6t^3 + 5t^2 + 9) \div (2t + 3)$

40. $(2h^3 - 5h^2 + 22h) \div (2h + 3)$

41. $\dfrac{9d^3 + 5d - 8}{3d - 2}$

42. $\dfrac{4x^3 + 5x^2 - 3x - 1}{4x + 1}$

43. $\dfrac{2x^4 + 3x^3 - 2x^2 - 3x - 6}{2x + 3}$

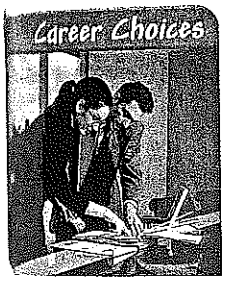
44. $\dfrac{6x^4 + 5x^3 + x^2 - 3x + 1}{3x + 1}$

★ 45. $\dfrac{x^3 - 3x^2 + x - 3}{x^2 + 1}$

★ 46. $\dfrac{x^4 + x^2 - 3x + 5}{x^2 + 2}$

★ 47. $\dfrac{x^3 + 3x^2 + 3x + 2}{x^2 + x + 1}$

★ 48. $\dfrac{x^3 - 4x^2 + 5x - 6}{x^2 - x + 2}$



Cost Analyst

Cost analysts study and write reports about the factors involved in the cost of production.

Online Research

For information about a career in cost analysis, visit: www.algebra2.com/careers

49. What is $x^3 - 2x^2 + 4x - 3$ divided by $x - 1$? $x^2 - x + 3$

50. Divide $2y^3 + y^2 - 5y + 2$ by $y + 2$. $2y^2 - 3y + 1$

51. **BUSINESS** A company estimates that it costs $0.03x^2 + 4x + 1000$ dollars to produce x units of a product. Find an expression for the average cost per unit.

52. **ENTERTAINMENT** A magician gives these instructions to a volunteer.

- Choose a number and multiply it by 3.
- Then add the sum of your number and 8 to the product you found.
- Now divide by the sum of your number and 2.

What number will the volunteer always have at the end? Explain.
4; See margin for explanation.

MEDICINE **For Exercises 53 and 54, use the following information.**
The number of students at a large high school who will catch the flu during an outbreak can be estimated by $n = \frac{170t^2}{t^2 + 1}$, where t is the number of weeks from the beginning of the epidemic and n is the number of ill people.

53. Perform the division indicated by $\frac{170t^2}{t^2 + 1}$. $170 - \frac{170}{t^2 + 1}$

54. Use the formula to estimate how many people will become ill during the first week. **85 people**

PHYSICS **For Exercises 55–57, suppose an object moves in a straight line so that after t seconds, it is $t^3 + t^2 + 6t$ feet from its starting point.** **55.** $x^3 + x^2 + 6x - 24$ ft

55. Find the distance the object travels between the times $t = 2$ and $t = x$.

56. How much time elapses between $t = 2$ and $t = x$? $x - 2$ s

57. Find a simplified expression for the average speed of the object between times $t = 2$ and $t = x$. $x^2 + 3x + 12$ ft/s

51. $\$0.03x + 4 + \dfrac{1000}{x}$

Pages 237–238, Lesson 5-3

21. $b^2 + 10b$

22. $x - 15$

23. $n^2 - 2n + 3$

24. $2c^2 + c + 5 + \dfrac{6}{c - 2}$

25. $x^3 - 5x^2 + 11x - 22 + \dfrac{39}{x + 2}$

26. $6w^4 + 12w^3 + 24w^2 + 30w + 60$

27. x^2

28. $x^2 + 3x + 9$

29. $y^2 - y - 1$

30. $m^2 - 7$

31. $a^3 - 6a^2 - 7a + 7 + \dfrac{3}{a + 1}$

32. $2m^3 + m^2 + 3m - 1 + \dfrac{5}{m - 3}$

33. $x^4 - 3x^3 + 2x^2 - 6x + 19 - \dfrac{56}{x + 3}$

34. $3c^4 - c^3 + 2c^2 - 4c + 9 - \dfrac{13}{c + 2}$

35. $g + 5$

36. $2b^2 - b - 1 + \dfrac{4}{b + 1}$

37. $t^4 + 2t^3 + 4t^2 + 5t + 10$

38. $y^4 - 2y^3 + 4y^2 - 8y + 16$

39. $3t^2 - 2t + 3$

40. $h^2 - 4h + 17 - \dfrac{51}{2h + 3}$

41. $3d^2 + 2d + 3 - \dfrac{2}{3d - 2}$

42. $x^2 + x - 1$

43. $x^3 - x - \dfrac{6}{2x + 3}$

Find $[g \circ h](x)$ and $[h \circ g](x)$. Justify each step. **See students' work.**

29. $g(x) = 4x$ **$8x - 4$;** $h(x) = 2x - 1$ **$8x - 1$**

30. $g(x) = -5x$ **$15x - 5$;** $h(x) = -3x + 1$ **$15x + 1$**

31. $g(x) = x + 2$ **$x^2 + 2$;** $h(x) = x^2$ **$x^2 + 4x + 4$**

34. $2x^2 - 5x + 9$; $2x^2 - x + 5$

32. $g(x) = x - 4$, $h(x) = 3x^2$

33. $g(x) = 2x$, $h(x) = x^3 + x^2 + x + 1$

34. $g(x) = x + 1$, $h(x) = 2x^2 - 5x + 8$

32. $3x^2 - 4$; $3x^2 - 24x + 48$ 33. $2x^3 + 2x^2 + 2x + 2$; $8x^3 + 4x^2 + 2x + 1$

If $f(x) = 4x$, $g(x) = 2x - 1$, and $h(x) = x^2 + 1$, find each value.

35. $f[g(-1)]$ **-12**

36. $h[g(4)]$ **50**

37. $g[f(5)]$ **39**

38. $f[h(-4)]$ **68**

39. $g[g(7)]$ **25**

40. $f[f(-3)]$ **-48**

41. $h\left[f\left(\frac{1}{4}\right)\right]$ **2**

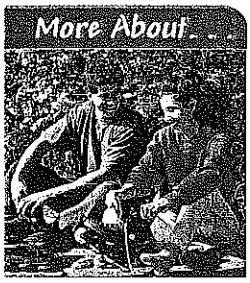
42. $g\left[h\left(-\frac{1}{2}\right)\right]$ **$1\frac{1}{2}$**

43. $[g \circ (f \circ h)](3)$ **79**

44. $[f \circ (h \circ g)](3)$ **104**

45. $[h \circ (g \circ f)](2)$ **226**

46. $[f \circ (g \circ h)](2)$ **36**



Shopping
Americans spent over $500 million on inline skates and equipment in 2000.
Source: National Sporting Goods Association

POPULATION GROWTH **For Exercises 47 and 48, use the following information.**
From 1990 to 1999, the number of births $b(x)$ in the U.S. can be modeled by the function $b(x) = -27x + 4103$, and the number of deaths $d(x)$ can be modeled by the function $d(x) = 23x + 2164$, where x is the number of years since 1990 and $b(x)$ and $d(x)$ are in thousands.

47. The net increase in population P is the number of births per year minus the number of deaths per year or $P = b - d$. Write an expression that can be used to model the population increase in the U.S. from 1990 to 1999 in function notation. **$P(x) = -50x + 1939$**

48. Assume that births and deaths continue at the same rates. Estimate the net increase in population in 2010. **939,000**

SHOPPING **For Exercises 49–51, use the following information.**
Liluye wants to buy a pair of inline skates that are on sale for 30% off the original price of $149. The sales tax is 5.75%.

49. Express the price of the inline skates after the discount and the price of the inline skates after the sales tax using function notation. Let x represent the price of the inline skates, $p(x)$ represent the price after the 30% discount, and $s(x)$ represent the price after the sales tax. **$p(x) = 0.70x$; $s(x) = 1.0575x$**

50. $s[p(x)]$; The 30% would be taken off first, and then the sales tax would be calculated on this price.

50. Which composition of functions represents the price of the inline skates, $p[s(x)]$ or $s[p(x)]$? Explain your reasoning.

51. How much will Liluye pay for the inline skates? **$110.30**

TEMPERATURE **For Exercises 52–54, use the following information.**
There are three temperature scales: Fahrenheit (°F), Celsius (°C), and Kelvin (K). The function $K(C) = C + 273$ can be used to convert Celsius temperatures to Kelvin. The function $C(F) = \frac{5}{9}(F - 32)$ can be used to convert Fahrenheit temperatures to Celsius.

52. Write a composition of functions that could be used to convert Fahrenheit temperatures to Kelvin. **$[K \circ C](F) = \frac{5}{9}(F - 32) + 273$**

53. Find the temperature in Kelvin for the boiling point of water and the freezing point of water if water boils at 212°F and freezes at 32°F. **373 K; 273 K**

54. While performing an experiment, Kimi found the temperature of a solution at different intervals. She needs to record the change in temperature in degrees Kelvin, but only has a thermometer with a Fahrenheit scale. What will she record when the temperature of the solution goes from 158°F to 256°F? **309.67 K**

55. **FINANCE** Kachina pays $50 each month on a credit card that charges 1.6% interest monthly. She has a balance of $700. The balance at the beginning of the nth month is given by $f(n) = f(n - 1) + 0.016 f(n - 1) - 50$. Find the balance at the beginning of the first five months. No additional charges are made on the card. (*Hint:* $f(1) = 700$) **$700, $661.20, $621.78, $581.73, $541.04**