Honors Geometry
Unit 7 Review: Similarity

Name: Key

Problems 1 - 5: Always, Sometimes, or Never

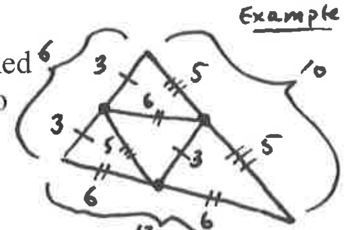
1. If two polygons are similar, then the ratio of their Perimeters is equal to the ratio of their corresponding sides. → Always! We proved it ☺ — A

2. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{(a+2b)}{b} = \frac{(c+2d)}{d}$. — A

$d(a+2b) = b(c+2d)$
$da + \cancel{2db} = bc + \cancel{2bd}$
$-2db \quad -2db$
$ad = bc \longleftrightarrow da = bc$ (same ✓)

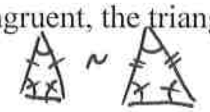
3. If the midpoints of the sides of a triangle are joined to form another triangle, this triangle is similar to the original triangle. Recall that in a △ if midpts are constructed, then perimeter of smaller △ is half the larger △. — A

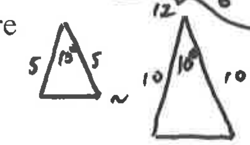
Example!



$\therefore \frac{6}{3} = \frac{10}{5} = \frac{12}{6}$
$\downarrow \quad \downarrow \quad \downarrow$
$2 \quad 2 \quad 2$
$\therefore$ △'s ~ by ~SSS

4. If the vertex angles of two isosceles triangles are congruent, the triangles are similar. — A





By ~AA, ~SAS, ...

5. If two triangles are congruent, then they are similar. — A

Always! Ex: △ABC ≅ △XYZ → similar ☺

6. If $10y = 4x$, then find the ratio of x to y. — $\frac{10}{4} = \boxed{\frac{5}{2}}$

$\frac{10y}{4y} = \frac{4x}{4y}$

means $\frac{x}{y}$

7. Find the fourth proportional to 3, 4, and 8. — $\boxed{\frac{32}{3}}$ or $\boxed{10\frac{2}{3}}$ or $\boxed{10.\overline{6}}$

$\frac{3}{4} = \frac{8}{x} \Rightarrow \frac{3x}{3} = \frac{32}{3} \Rightarrow x = \frac{32}{3}$ (1st, 2nd, 3rd, 4th)

8. Find the mean proportional between 8 and 18. — $\boxed{\pm 12}$

Geometric mean

$\frac{8}{x} = \frac{x}{18} \Rightarrow \sqrt{x^2} = \sqrt{144}$
$\Rightarrow x = \pm 12$ ✓

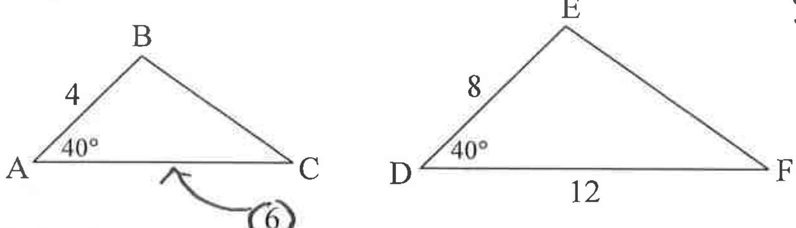
9. State another name for "mean proportional". — Geometric mean

Geometric mean

10. Why is $\Delta ABC \sim \Delta DEF$? — ~SAS

$\frac{AB}{DE}$ $\quad \frac{AC}{DF}$

$\frac{4}{8} = \frac{1}{2}$, $\angle A \cong \angle D$, $\frac{6}{12} = \frac{1}{2}$



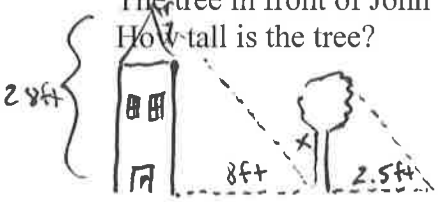
11. Solve for x. — $\boxed{-\frac{5}{4}}$

$$\frac{(x-4)}{(x+5)} = \frac{(x+3)}{x}$$

$x(x-4) = (x+5)(x+3)$
$x^2 - 4x = x^2 + 3x + 5x + 15$
$\cancel{x^2} - 4x = \cancel{x^2} + 8x + 15$
$-x^2 \quad -x^2$
$-4x = 8x + 15$

$-4x = \cancel{8x} + 15$
$-8x \quad \cancel{-8x}$
$-12x = 15$
$\frac{-12x}{-12} = \frac{15}{-12}$
$x = -\frac{15}{12} = -\frac{5}{4}$ ✓

12. John's house is 28 feet tall and it casts an 8 foot shadow. The tree in front of John's house casts a 2.5 foot shadow. How tall is the tree?



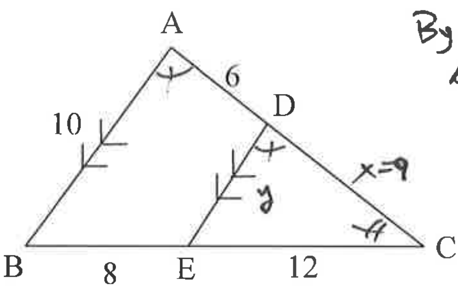
$$\frac{\text{Height building}}{\text{Height tree}} = \frac{\text{Building shadow}}{\text{Tree shadow}}$$

$$\frac{28}{x} = \frac{8}{2.5} \Rightarrow \frac{8x}{8} = \frac{70}{8} \Rightarrow x = 8.75$$ ✓

$\boxed{8.75\text{ft}}$ Height of tree

Use the diagram below for question 13 and 14.

13. Find DE.



By definition of similarity:
$\triangle CDE \sim \triangle CAB \Rightarrow$

$$\frac{CD}{CA} = \frac{DE}{AB} = \frac{CE}{CB}. \quad \text{So } \frac{9}{15} = \frac{y}{10}$$

$15y = 90 \Rightarrow y = 6$ ✓

$\boxed{y=6}$

14. Find DC.

By side splitter, $\frac{CD}{DA} = \frac{CE}{EB} \Rightarrow \frac{x}{6} = \frac{12}{8} \Rightarrow 8x = 72 \Rightarrow x = 9$ ✓

$\boxed{DC=9}$

15. Given: $\triangle PQR \sim \triangle BAC \Rightarrow \frac{PQ}{BA} = \frac{QR}{AC} = \frac{PR}{BC}$

$AB = 6$, $PR = 12$
$PQ = 9$, $QR = 15$

$$\boxed{\frac{9}{6} = \frac{15}{AC}} = \frac{12}{BC}$$

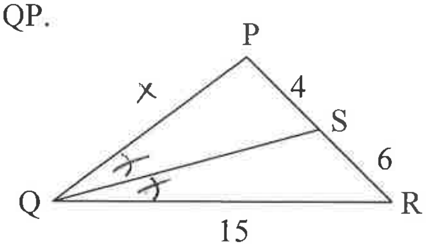
Find AC.

$$\frac{9 \cdot AC}{9} = \frac{90}{9} \qquad AC = 10$$ ✓

$\boxed{AC=10}$

16. Given: QS bisects $\angle PQR$ ⇒ Angle Bisector Theorem ⇒

Find QP.

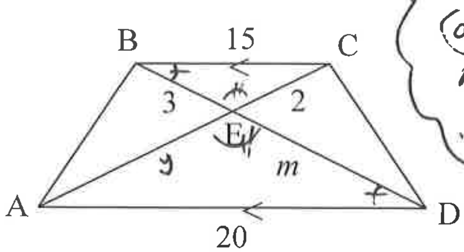


$$\frac{RS}{SP} = \frac{QR}{QP}$$

$$\frac{6}{4} = \frac{15}{x} \Rightarrow \frac{6x}{6} = \frac{60}{6} \Rightarrow x = 10$$ ✓

$\boxed{x=10}$

17. Find m.



$\overline{BC} \parallel \overline{AD}$ in trap BCDA ⇒
(alt int. ∠s) $\angle CBD \cong \angle ADB$.
Also vertical ∠'s ≅, so $\angle BEC \cong \angle DEA$
∴ $\triangle CBE \sim \triangle ADE \Rightarrow \frac{CB}{AD} = \frac{BE}{DE} = \frac{CE}{AE}$

Don't have to show... just to make you understand

$$\Rightarrow \boxed{\frac{15}{20} = \frac{3}{m}} = \frac{2}{y}$$

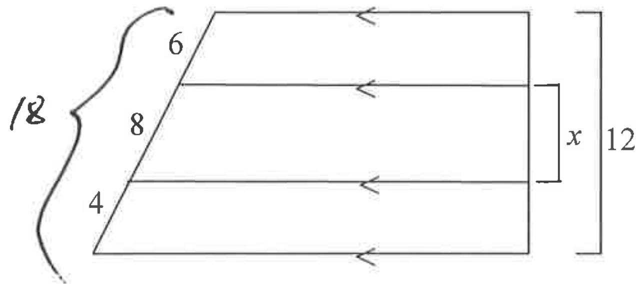
$$\frac{15m}{15} = \frac{60}{15}$$

$m = 4$ ✓

$\boxed{m=4}$

$\boxed{\frac{16}{3}}$ or $\boxed{5\frac{1}{3}}$ or $\boxed{5.\overline{3}}$

18. Find *x*.



$$\frac{8}{18} \times \frac{x}{12}$$

$$\frac{18x}{18} = \frac{96}{18}$$

$$x = \frac{96 \div 6}{18 \div 6} = \frac{16}{3}$$

19. If $\dfrac{2}{(x+4)} = \dfrac{3}{(y+6)}$, find $y : x$.

$\boxed{\frac{3}{2}}$

$2(y+6) = 3(x+4)$

$y : x \Rightarrow \frac{y}{x}$

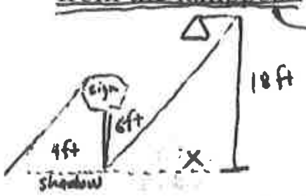
$2y + \cancel{12} = 3x + \cancel{12}$

$-12 \qquad -12$

$\frac{\cancel{2}y}{\cancel{2}x} = \frac{3\cancel{x}}{2\cancel{x}} \Rightarrow \frac{y}{x} = \frac{3}{2}$ ✓

20. The light from an 18 foot street light creates a 4 foot shadow in front of a 6 foot sign. How far is the sign from the lamppost? → x

$\boxed{12 ft}$



$$\frac{4}{x} \times \frac{6}{18}$$

$$\frac{6x}{6} = \frac{72}{6} \Rightarrow x = 12$$

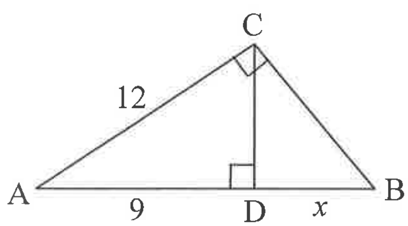
21. Given: $\Delta ABC \sim \Delta ACD$
AD = 9, AC = 12

$\boxed{x=7}$

$$\frac{AB}{AC} = \frac{BC}{CD} = \frac{AC}{AD}$$

$$\boxed{\frac{9+x}{12}} = \frac{BC}{CD} = \boxed{\frac{12}{9}}$$

Find AB.



$$\frac{(9+x)}{12} \times \frac{12}{9} \Rightarrow$$

$9(9+x) = 144$

$81 + 9x = 144$

$-81 \qquad -81$

$\frac{9x}{9} = \frac{63}{9} \Rightarrow x = 7$

22. Given: ABDG is a parallelogram ⇒ Alt. int. ∠'s ≅
⇒ opp. sides are ≅
$\overline{BC} \parallel \overline{EF}$
AB = 10 ED = 2.7
GF = 7 BC = $\frac{5}{3}$

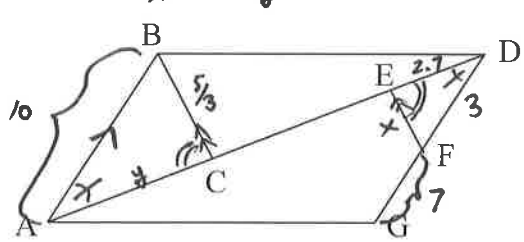
$\boxed{y = 3 = AC}$

$\boxed{x = \frac{1}{2}}$

∠BAD ≅ ∠GDA

Find EF and AC.
x y

Alt. ~~ext.~~ ∠s ≅, ∠BCA ≅ ∠FED



∴ $\Delta ACB \sim \Delta DEF$. So

$$\frac{AC}{DE} = \frac{CB}{EF} = \frac{AB}{DF}$$

$$\frac{y}{2.7} = \frac{5/3}{x} = \frac{10}{3}$$

1st: $\frac{y}{2.7} \times \frac{10}{3} \Rightarrow 3y = 27 \Rightarrow y = 3$

2nd: $\frac{5/3}{x} \times \frac{10}{3} \Rightarrow \frac{5}{3} \cdot 3 = 10x$

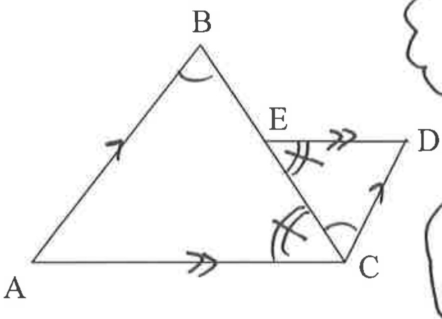
$\Rightarrow \frac{15}{3} = 10x$ $\quad x = \frac{5}{10} = \frac{1}{2}$

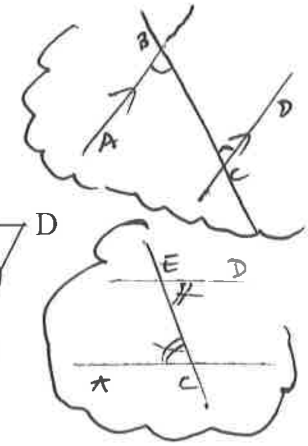
$\Rightarrow \frac{5}{10} = \frac{10}{10}x$

23. Given: $\overline{AB} \parallel \overline{DC}$

$\overline{ED} \parallel \overline{AC}$

Prove: $AB \cdot CE = DC \cdot BC$





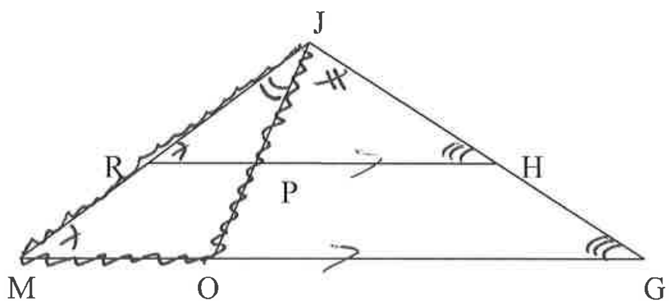
Picture you should always draw if confused about how to write ∠s.

Statement	Reason
① $\overline{AB} \parallel \overline{DC}$	① Given
② $\overline{ED} \parallel \overline{AC}$	② Given
③ $\angle ABC \cong \angle DCE$	③ $\parallel$ lines $\Rightarrow$ Alternate interior ∠'s ≅
④ $\angle ACB \cong \angle DEC$	④ same as step 3
⑤ $\triangle ACB \sim \triangle DEC$	⑤ ~AA (3, 4)
⑥ $\frac{AC}{DE} = \boxed{\frac{CB}{EC} = \frac{AB}{DC}}$	⑥ Corresponding sides in ~ Δs are proportional
⑦ $CB \cdot DC = EC \cdot AB$	⑦ Means-Extremes Product Theorem

Notice same as $AB \cdot CE = DC \cdot BC$

24. Given: $\overline{HR} \parallel \overline{GM}$

Prove: $(PR)(OG) = (PH)(OM)$



Statement	Reason
① $\overline{HR} \parallel \overline{GM}$	① Given
② $\angle JRP \cong \angle JMO$	② $\parallel$ lines $\Rightarrow$ corr. ∠'s ≅
③ $\angle MJO \cong \angle RJP$	③ Reflexive Property of ∡s
④ $\triangle PRJ \sim \triangle OMJ$	④ ~AA
⑤ $\boxed{\frac{PR}{OM} = \frac{RJ}{MJ} = \frac{PJ}{OJ}}$	⑤ Corresponding sides of ~ Δs are proportional
⑥ $\angle PHJ \cong \angle OGJ$	⑥ Same as step 2
⑦ $\angle OJG \cong \angle PJH$	⑦ Same as step 3
⑧ $\triangle PHJ \sim \triangle OGJ$	⑧ same as step 4
⑨ $\boxed{\frac{PH}{OG} = \frac{HJ}{GJ} = \frac{PJ}{OJ}}$	⑨ Same as step 5
⑩ $\frac{PR}{OM} = \frac{PH}{OG}$	⑩ Transitive Property (step 5, 9)
⑪ $(PR)(OG) = (OM)(PH)$	⑪ Means Extremes Product Theorem

Done