PA 7-1 p. 641 #5-23 odd

Sample Space (set of all possible outcomes)

(5) 4 candidates for homecoming queen
3 candidates for homecoming king

$\boxed{4 \times 3 = 12 \text{ possible king-queen pairs}}$

	Q_1	Q_2	Q_3	Q_4
K_1	K_1Q_1	K_1Q_2	K_1Q_3	K_1Q_4
K_2	K_2Q_1	K_2Q_2	K_2Q_3	K_2Q_4
K_3	K_3Q_1	K_3Q_2	K_3Q_3	K_3Q_4

(7) LOGARITHM
(Notice no letters repeat)
9 letters

$\therefore, 9! = 9\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1 = \boxed{362,880}$

(ALGORITHM)

(9) MISSISSIPPI

$\left.\begin{array}{l}\text{11 letters}\\ \text{4 I's}\\ \text{4 S's}\\ \text{2 P's}\end{array}\right\}$ $\therefore, \dfrac{11!}{4!\cdot4!\cdot2!} = \boxed{34,650}$

(11) We have 3 distinguishable positions:
President, VP, Secretary ...
(order matters)

Put people in Particular Positions

$n = 13$
$r = 3$

$${}_{13}P_3 = \frac{13!}{(13-3)!} = \frac{13\cdot12\cdot11\cdot\cancel{10!}}{\cancel{10!}} = 13\cdot12\cdot11 = \boxed{1,716}$$

(13) $4! = 4\cdot3\cdot2\cdot1 = \boxed{24}$

(15) $${}_6P_2 = \frac{6!}{(6-2)!} = \frac{6\cdot5\cdot\cancel{4!}}{\cancel{4!}} = 6\cdot5 = \boxed{30}$$

(17) $${}_{10}C_7 = \binom{10}{7} = \frac{10!}{7!(10-7)!} = \frac{10!}{7!\cdot3!} = \frac{10\cdot9\cdot8\cdot\cancel{7!}}{\cancel{7!}\cdot3!} = \frac{10\cdot\cancel{9}^{3}\cdot\cancel{8}^{4}}{\cancel{3}\cdot\cancel{2}\cdot1} = \boxed{120}$$

(19) 13 cards are selected from a standard deck of 52 cards to form a $\underbrace{\text{bridge}}_{\text{card game}}$ hand.

$\boxed{\text{Combination}}$

(order of cards NOT important in hand)

(21) 4 students selected from senior class to form committee

$\boxed{\text{Combination}}$

(order NOT important)

(23) PLATE : $\underbrace{\_\ \_}_{\text{digits}}\ \underbrace{\_\ \_}_{\text{letters}}\ \underbrace{\_\ \_\ \_}_{\text{digits}}$

0, 1, 2, 3, 4, 5, 6, 7, 8, 9 (10 possible digits)
A, B, C, D, ..., X, Y, Z (26 letters in alphabet)

No letters/digits repeat (without replacement)

$\therefore, \underline{10}\cdot\underline{9}\cdot\underline{26}\cdot\underline{25}\cdot\underline{8}\cdot\underline{7}\cdot\underline{6} = \boxed{19,656,000}$

PA 7-2 p. 641 # 25 – 41 odd

(25) 2 dice rolled (1 red and 1 green)

Six-sided die:

So when 2 dice are rolled:

$6 \times 6 = \boxed{36 \text{ possible outcomes}}$

Sample space

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$\boxed{36 \text{ possible outcomes}}$

(27) order NOT important, so combination (choosing 3 from 25)

$${}_{25}C_3 = \binom{25}{3} = \frac{25!}{3!(25-3)!} = \frac{25!}{3! \cdot 22!} = \frac{25 \cdot 24 \cdot 23 \cdot \cancel{22!}}{3 \cdot 2 \cdot 1 \cdot \cancel{22!}} = \boxed{2{,}300 \text{ committees}}$$

(29) order NOT important, so combination (choosing 3 from 48)

$${}_{48}C_3 = \binom{48}{3} = \frac{48!}{3!(48-3)!} = \frac{48!}{3! \cdot 45!} = \frac{48 \cdot 47 \cdot 46 \cdot \cancel{45!}}{3 \cdot 2 \cdot 1 \cdot \cancel{45!}} = \boxed{17{,}296 \text{ different sets of discs}}$$

(31) How many different 13-card hands include ace of spades AND king of spades?

✓ Combination

✓ 52 cards in a standard deck

Need: Ace of spades AND King of spades, so we have $13 - 2 = 11$ cards still needing to be chosen from $52 - 2 = 50$ cards remaining.

$$\therefore, \ {}_{50}C_{11} \cdot {}_2C_2 = \binom{50}{11}\binom{2}{2} = \left(\frac{50!}{11!(50-11)!}\right)\left(\frac{2!}{2!(2-2)!}\right) = \left(\frac{50!}{11! \cdot 39!}\right)\left(\frac{\cancel{2!}}{\cancel{2!} \cdot 0!}\right)$$

Recall: $0! = 1$

$$= \frac{50!}{11! \cdot 39!} \cdot 1$$

$$= \boxed{37{,}353{,}738{,}800}$$

(33) 6 seniors meet qualifications BUT university allows the school to nominate up to three students, AND the school always nominates at least one student. How many different choices could the nominating committee make?

(1) The school can (choose to) nominate 1 student

OR

(2) The school can (choose to) nominate 2 students

OR

(3) The school can (choose to) nominate 3 students

$${}_6C_1 = \binom{6}{1} = \frac{6!}{1!(6-1)!} = \frac{6!}{1! \cdot 5!} = \frac{6 \cdot \cancel{5!}}{1 \cdot \cancel{5!}} = 6$$

$${}_6C_2 = \binom{6}{2} = \frac{6!}{2!(6-2)!} = \frac{6!}{2! \cdot 4!} = \frac{6 \cdot 5 \cdot \cancel{4!}}{2 \cdot 1 \cdot \cancel{4!}} = 15$$

$${}_6C_3 = \binom{6}{3} = \frac{6!}{3!(6-3)!} = \frac{6!}{3! \cdot 3!} = \frac{6 \cdot 5 \cdot 4 \cdot \cancel{3!}}{3 \cdot 2 \cdot 1 \cdot \cancel{3!}} = 20$$

$\therefore$, Total # of different choices are: $6 + 15 + 20 = \boxed{41}$

(35) 6 possible outcomes for each of the 5 dice.

$\therefore,\ 6\cdot 6\cdot 6\cdot 6\cdot 6 = 6^5 = \boxed{7,776}$

(37) Mary takes equal amounts of salad she chooses
But she likes to vary selection
If she can choose from among 9 different salads, how many different lunches can she create?

Suppose Mary had (equal amounts of each):

(1) 1 salad (to choose from)

$\therefore$, $\boxed{\text{1 lunch possible}}$

(2) 2 salads (say A, B)

$\therefore$, A, B, AB

$\therefore$, $\boxed{\text{3 possible lunches}}$

(3) 3 salads (say A, B, C)

$\therefore$, A, B, C, AB, AC, BC, ABC

$\therefore$, $\boxed{\text{7 possible lunches}}$

(4) 4 salads (say A, B, C, D)

$\therefore$, A, B, C, D, AB, AC, AD, BC, BD, CD, ABC, ABD, BCD, CAD, ABCD

$\therefore$, $\boxed{\text{15 possible lunches}}$

• Do you see a pattern of # of possible lunches?

1, 3, 7, 15, ... ?

$2^1 - 1 = 1$
$2^2 - 1 = 3$
$2^3 - 1 = 7$
$2^4 - 1 = 15$
$\vdots$

$\therefore,\ 2^{\#\text{ of salads}} - 1 \stackrel{\wedge}{=} \#$ of possible lunches

$\therefore,\ 2^9 - 1 = \boxed{511}$

(39) Lurgi sells one size pizza
CLAIM: Selection of toppings "allows for more $>$ than 4000 choices"
What is the smallest # of toppings Lurgi can offer?

$2^{\#\text{ of topping}} = \#$ of pizzas

$\therefore,\ 2^{12} > 4000$
$4096 > 4000$

$\boxed{\therefore,\ \text{Lurgi offers at least 12 toppings}}$

(1) Suppose Lurgi offered 1 topping. Then you can have a pizza w/ that 1 topping or pizza without that topping.

$\therefore$, $\boxed{\text{2 pizzas}}$

(2) 2 toppings (say A, B)

No topping ← ○, A, B, AB

$\therefore$, $\boxed{\text{4 pizzas}}$

(3) 3 toppings (say A, B, C)

No topping ← ○, A, B, C, AB, AC, BC, ABC

$\therefore$, $\boxed{\text{8 pizzas}}$

• Do you notice a pattern?

2, 4, 8, ...

(41) # of possible answer key for T-F test?

1 question

Answer key

T or F

$\therefore$, $\boxed{\text{2 possibilities}}$

2 questions

Answer key

TT
TF
FT
FF

$\therefore$, $\boxed{\text{4 possibilities}}$

3 questions

Answer key

TTT FFT
TTF FTF
TFT TFF
FTT FFF

$\therefore$, $\boxed{\text{8 possibilities}}$

Do you see a pattern?
2, 4, 8 ...
$2^{\#\text{ quest}} = \#$ of poss. answer keys

$\boxed{\therefore,\ 2^{10} = 1024}$ ☒

PA7-3 p. 648 # 1-15 odd, 27, 28

① Using the Nspire: $C(4, \overbrace{\{0,1,2,3,4\}}^{r}) = \{1,4,6,4,1\}$

$$(a+b)^4 = \binom{4}{0}a^4b^0 + \binom{4}{1}a^3b^1 + \binom{4}{2}a^2b^2 + \binom{4}{3}a^1b^3 + \binom{4}{4}a^0b^4$$

$$= 1a^4(1) + 4a^3b + 6a^2b^2 + 4ab^3 + 1(1)b^4$$

$$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$\boxed{\therefore, (a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4}$$

③ $C(7, \{0,1,2,3,4,5,6,7\}) = \{1,7,21,35,35,21,7,1\}$

$$(x+y)^7 = \binom{7}{0}x^7y^0 + \binom{7}{1}x^6y^1 + \binom{7}{2}x^5y^2 + \binom{7}{3}x^4y^3 + \binom{7}{4}x^3y^4 + \binom{7}{5}x^2y^5 + \binom{7}{6}x^1y^6 + \binom{7}{7}x^0y^7$$

$$= x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$$

$$\boxed{\therefore, (x+y)^7 = x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7}$$

⑤

1

1 1

1 2 1

3rd row → $\boxed{1 \quad 3 \quad 3 \quad 1}$

$\therefore, (x+y)^3 = 1x^3y^0 + 3x^2y^1 + 3x^1y^2 + 1x^0y^3$

$$\boxed{(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3}$$

⑦

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

1 7 21 35 35 21 7 1

8th row → $\boxed{1 \quad 8 \quad 28 \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1}$

$\therefore, (p+q)^8 = 1p^8q^0 + 8p^7q^1 + 28p^6q^2 + 56p^5q^3 + 70p^4q^4 + 56p^3q^5 + 28p^2q^6 + 8p^1q^7 + 1p^0q^8$

$$\boxed{\therefore, (p+q)^8 = p^8 + 8p^7q + 28p^6q^2 + 56p^5q^3 + 70p^4q^4 + 56p^3q^5 + 28p^2q^6 + 8pq^7 + q^8}$$

⑨ $\binom{9}{2} = \dfrac{9!}{2!(9-2)!} = \dfrac{9!}{2!\cdot 7!} = \dfrac{9\cdot 8\cdot \cancel{7!}}{2!\cdot \cancel{7!}} = \dfrac{9\cdot \overset{4}{\cancel{8}}}{\cancel{2}\cdot 1} = \dfrac{36}{1} = \boxed{36}$ $\quad \boxed{\therefore, \binom{9}{2} = 36}$

⑪ $\binom{166}{166} = \dfrac{166!}{166!(166-166)!} = \dfrac{\cancel{166!}}{\cancel{166!}\cdot 0!} = \dfrac{1}{0!} = \dfrac{1}{1} = \boxed{1}$ $\quad \boxed{\therefore, \binom{166}{166} = 1}$

⑬ $x^{11}y^3$ term, $(x+y)^{14}$

Recall: Terms of expansion are of form: $\binom{n}{k}a^{\overbrace{n-k}^{\text{add to } n}}b^{k}$ same

$\therefore, \binom{\overset{14}{14}}{3}x^{11}y^{3}$

$\therefore$, Coefficient: $\binom{14}{3} = \dfrac{14!}{3!(14-3)!} = \dfrac{14!}{3!\cdot 11!} = \dfrac{14\cdot 13\cdot 12\cdot \cancel{11!}}{3!\cdot \cancel{11!}}$

$$= \frac{{}^{7}\cancel{14}\cdot 13\cdot \cancel{12}^{4}}{\cancel{3}\cdot \cancel{2}\cdot 1} = \boxed{364}$$

⑮ x^4 term, $(x-2)^{12}$

$\binom{\overset{12}{12}}{8}x^4(-2)^{8} = 495\cdot x^4\cdot 256$

$\therefore$, Coefficient: $495(256) = \boxed{126{,}720}$

(27) Answers vary depending on calculator used.

- TI-84 Plus Silver Edition can compute $\boxed{69!}$.
  70! results in OVERFLOW
- Nspire Calculator can compute $\boxed{449!}$.
  450! results in OVERFLOW

(28) $\binom{n}{100}$; Again answers vary depending on calculator used.

TI-Nspire : Largest value of $\boxed{n = 362{,}827{,}366{,}522}$

— Sergio Lara, 2018

PA 7-4 p. 648 # 17-25 odd, 29, 34-38 all, 40

(17) $f(x) = (x-2)^5$

$a = x$, $b = -2$, $n = 5$

$(x-2)^5 = \binom{5}{0}x^5(-2)^0 + \binom{5}{1}x^4(-2)^1 + \binom{5}{2}x^3(-2)^2 + \binom{5}{3}x^2(-2)^3 + \binom{5}{4}x^1(-2)^4 + \binom{5}{5}x^0(-2)^5$

$= 1\cdot x^5\cdot 1 + 5x^4(-2) + 10x^3(4) + 10x^2(-8) + 5x(16) + 1\cdot 1\cdot(-32)$

$= x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$

$\boxed{\therefore,\ f(x) = (x-2)^5 = x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32}$

(19) $h(x) = (2x-1)^7$

$a = 2x$, $b = -1$, $n = 7$

$(2x-1)^7 = \binom{7}{0}(2x)^7(-1)^0 + \binom{7}{1}(2x)^6(-1)^1 + \binom{7}{2}(2x)^5(-1)^2 + \binom{7}{3}(2x)^4(-1)^3 + \binom{7}{4}(2x)^3(-1)^4 + \binom{7}{5}(2x)^2(-1)^5 + \binom{7}{6}(2x)^1(-1)^6 + \binom{7}{7}(2x)^0(-1)^7$

$= 1\cdot 2^7x^7\cdot 1 + 7\cdot 2^6x^6(-1) + 21\cdot 2^5\cdot x^5\cdot 1 + 35\cdot 2^4x^4(-1) + 35\cdot 2^3x^3(1) + 21\cdot 2^2x^2(-1) + 7\cdot 2x(1) + 1\cdot 1\cdot(-1)$

$= 128x^7 - 448x^6 + 672x^5 - 560x^4 + 280x^3 - 84x^2 + 14x - 1$

$\boxed{\therefore,\ h(x) = (2x-1)^7 = 128x^7 - 448x^6 + 672x^5 - 560x^4 + 280x^3 - 84x^2 + 14x - 1}$

(21) $(2x+y)^4 = \binom{4}{0}(2x)^4y^0 + \binom{4}{1}(2x)^3(y)^1 + \binom{4}{2}(2x)^2(y)^2 + \binom{4}{3}(2x)^1(y)^3 + \binom{4}{4}(2x)^0(y)^4$

$a = 2x$, $b = y$, $n = 4$

$= 1\cdot 2^4\cdot x^4\cdot 1 + 4\cdot 2^3x^3\cdot y + 6\cdot 2^2x^2y^2 + 4\cdot 2x\cdot y^3 + 1\cdot 1\cdot y^4$

$= 16x^4 + 32x^3y + 24x^2y^2 + 8xy^3 + y^4$

$\boxed{\therefore,\ (2x+y)^4 = 16x^4 + 32x^3y + 24x^2y^2 + 8xy^3 + y^4}$

(23) $(\sqrt{x} - \sqrt{y})^6 = \binom{6}{0}(\sqrt{x})^6(-\sqrt{y})^0 + \binom{6}{1}(\sqrt{x})^5(-\sqrt{y})^1 + \binom{6}{2}(\sqrt{x})^4(-\sqrt{y})^2 + \binom{6}{3}(\sqrt{x})^3(-\sqrt{y})^3 + \binom{6}{4}(\sqrt{x})^2(-\sqrt{y})^4 + \binom{6}{5}(\sqrt{x})^1(-\sqrt{y})^5 + \binom{6}{6}(\sqrt{x})^0(-\sqrt{y})^6$

$a = \sqrt{x}$, $b = -\sqrt{y}$, $n = 6$

Recall: $\sqrt[2]{x} = x^{\frac{1}{2}}$

$= 1\cdot x^{\frac{6}{2}}\cdot 1 + 6x^{\frac{5}{2}}(-\sqrt{y}) + 15x^{\frac{4}{2}}(+y^{\frac{2}{2}}) + 20x^{\frac{3}{2}}(-y^{\frac{3}{2}}) + 15x^{\frac{2}{2}}(+y^{\frac{4}{2}}) + 6\sqrt{x}(-y^{\frac{5}{2}}) + 1\cdot 1(+y^{\frac{6}{2}})$

$= x^3 - 6x^{\frac{5}{2}}y^{\frac{1}{2}} + 15x^2y - 20x^{\frac{3}{2}}y^{\frac{3}{2}} + 15xy^2 - 6x^{\frac{1}{2}}y^{\frac{5}{2}} + y^3$

$\boxed{\therefore,\ (\sqrt{x} - \sqrt{y})^6 = x^3 - 6x^{5/2}y^{1/2} + 15x^2y - 20x^{3/2}y^{3/2} + 15xy^2 - 6x^{1/2}y^{5/2} + y^3}$

or

$\boxed{(\sqrt{x} - \sqrt{y})^6 = x^3 - 6\sqrt{x^5y} + 15x^2y - 20\sqrt{x^3y^3} + 15xy^2 - 6\sqrt{xy^5} + y^3}$

(25) $(x^{-2}+3)^5 = \binom{5}{0}(x^{-2})^5(3)^0 + \binom{5}{1}(x^{-2})^4(3)^1 + \binom{5}{2}(x^{-2})^3(3)^2 + \binom{5}{3}(x^{-2})^2(3)^3 + \binom{5}{4}(x^{-2})^1(3)^4 + \binom{5}{5}(x^{-2})^0(3)^5$

$a = x^{-2}$, $b = 3$, $n = 5$

$= 1\cdot x^{-10}\cdot 1 + 5x^{-8}\cdot 3 + 10x^{-6}\cdot 9 + 10x^{-4}(27) + 5x^{-2}(81) + 1\cdot 1\cdot 243$

$= x^{-10} + 15x^{-8} + 90x^{-6} + 270x^{-4} + 405x^{-2} + 243$

$\boxed{\therefore,\ (x^{-2}+3)^5 = x^{-10} + 15x^{-8} + 90x^{-6} + 270x^{-4} + 405x^{-2} + 243}$

(29) Prove $\binom{n}{1} = \binom{n}{n-1} = n$ $\forall$ integers $n \geq 1$.

Pf: $\binom{n}{1} = \dfrac{n!}{1!(n-1)!} = \dfrac{n\cancel{(n-1)!}}{1!\cancel{(n-1)!}} = \dfrac{n}{1} = n$ ✓

$\binom{n}{n-1} = \dfrac{n!}{(n-1)!(n-(n-1))!} = \dfrac{n!}{(n-1)!(\cancel{n}-\cancel{n}+1)!} = \dfrac{n!}{(n-1)!\cdot 1!} = \dfrac{n\cancel{(n-1)!}}{\cancel{(n-1)!}\cdot 1} = \dfrac{n}{1} = n$ ✓

$\Big\}$ $\therefore,\ \binom{n}{1} = \binom{n}{n-1}$ also.

(34) $\binom{n}{n-2} + \binom{n+1}{n-1} = n^2 \quad \forall$ integers $n \geq 2$.

Pf:

$$\binom{n}{n-2} + \binom{n+1}{n-1} = \frac{n!}{(n-2)![n-(n-2)]!} + \frac{(n+1)!}{(n-1)![(n+1)-(n-1)]!}$$

$$= \frac{n!}{(n-2)![\cancel{n}-\cancel{n}+2]!} + \frac{(n+1)!}{(n-1)![\cancel{n}+1-\cancel{n}+1]!}$$

$$= \frac{n!}{(n-2)!\cdot 2!} + \frac{(n+1)!}{(n-1)!\cdot 2!}$$

$$= \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}\cdot 2!} + \frac{(n+1)[\cancel{(n+1)-1}][(n+1)-2]!}{(n-1)!\cdot 2!}$$

$$= \frac{n(n-1)}{2!} + \frac{(n+1)(n)\cancel{(n-1)!}}{\cancel{(n-1)!}\cdot 2!} \qquad 2! = 2\cdot 1 = 2$$

$$= \frac{n(n-1)}{2} + \frac{n(n+1)}{2}$$

$$= \frac{n^2 \cancel{-n} + n^2 \cancel{+n}}{2}$$

$$= \frac{\cancel{2}n^2}{\cancel{2}}$$

$$= n^2 \checkmark$$

$\therefore \binom{n}{n-2} + \binom{n+1}{n-1} = n^2$.

(35) The coefficients in the polynomial expansion of $(x-y)^{50}$ alternate in sign. Justify.

$\boxed{\text{TRUE. Powers of } -y \text{ alternate between even and odd. Odd powers (eg. } (-y)^3 = \underbrace{(-y)(-y)}_{+}(-y) = y^2(-y) = -y^3\text{) will result in having negative coefficients.}}$

(36) The sum of any row of Pascal's Triangle is an even integer. Justify.

TRUE (starting with row 1 and ignoring row 0)

$$\begin{array}{l}
1 \\
1 \quad 1 \longrightarrow \text{row 1 sum}: 2 = 2^1 \\
1 \quad 2 \quad 1 \longrightarrow \text{row 2 sum}: 4 = 2^2 \\
1 \quad 3 \quad 3 \quad 1 \longrightarrow \text{row 3 sum}: 8 = 2^3 \\
1 \quad 4 \quad 6 \quad 4 \quad 1 \longrightarrow \text{row 4 sum}: 16 = 2^4 \\
1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \longrightarrow \text{row 5 sum}: 32 = 2^5 \\
1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1 \longrightarrow \text{row 6 sum}: 64 = 2^6 \\
1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1 \longrightarrow \text{row 7 sum}: 128 = 2^7 \\
\vdots \qquad\qquad \ddots \quad \text{row } n \text{ sum}: 2^n
\end{array}$$

(37) Coefficient of x^4 term in expansion of $(2x+1)^8$

a = 2x
b = 1
n = 8

$\binom{8}{\boxed{4}}(2x)^4(1)^{\boxed{4}}$ Must be the same

$= \binom{8}{4}2^4x^4(1)^4$ ∴, Coefficient: $\binom{8}{4}(16) = 70(16) = \boxed{1120, (C)}$

$= \binom{8}{4}(16)x^4(1)$

(38) Using the Nspire:

$C(10, \{0,1,2,3,4,5,6,7,8,9,10\}) = \{1, 10, 45, 120, 210, 252, 210, 120, 45, 10, 1\}$

∴, $\boxed{5}$ does <u>NOT</u> appear in 10^{th} row of Pascal's Δ. $\boxed{∴, (B)}$

(40) $(x+y)^3 + (x-y)^3 = ?$ Apply Binomial Theorem

$(x+y)^3 = \binom{3}{0}x^3y^0 + \binom{3}{1}x^2y^1 + \binom{3}{2}x^1y^2 + \binom{3}{3}x^0y^3$

a=x
b=y
n=3

$= 1 \cdot x^3 \cdot 1 + 3x^2y + 3xy^2 + 1 \cdot 1 \cdot y^3$

$= x^3 + 3x^2y + 3xy^2 + y^3$

$(x-y)^3 = \binom{3}{0}x^3(-y)^0 + \binom{3}{1}x^2(-y)^1 + \binom{3}{2}x^1(-y)^2 + \binom{3}{3}x^0(-y)^3$

a=x
b=-y
n=3

$= 1 \cdot x^3 \cdot 1 + 3x^2(-y) + 3x(-y)^2 + 1 \cdot 1 \cdot (-y)^3$ $(-y)(-y) = y^2$

$= x^3 - 3x^2y + 3xy^2 - y^3$

∴, $(x+y)^3 + (x-y)^3 = x^3 + \cancel{3x^2y} + 3xy^2 + \cancel{y^3} + x^3 - \cancel{3x^2y} + 3xy^2 - \cancel{y^3}$

$= 2x^3 + 6xy^2$

$\boxed{∴, (D)}$

PA 7-5 p. 656 # 1-9 odd, 21, 23, 29

① $u_n = \frac{n+1}{n}$

First six terms:

$u_1 = \frac{1+1}{1} = \frac{2}{1} = \boxed{2}$

$u_2 = \frac{2+1}{2} = \boxed{\frac{3}{2}}$

$u_3 = \frac{3+1}{3} = \boxed{\frac{4}{3}}$

$u_4 = \frac{4+1}{4} = \boxed{\frac{5}{4}}$

$u_5 = \frac{5+1}{5} = \boxed{\frac{6}{5}}$

$u_6 = \frac{6+1}{6} = \boxed{\frac{7}{6}}$

100th term:

$u_{100} = \frac{100+1}{100} = \boxed{\frac{101}{100}}$

③ $c_n = n^3 - n$

First six terms:

$c_1 = (1)^3 - 1 = 1 - 1 = \boxed{0}$

$c_2 = (2)^3 - 2 = 8 - 2 = \boxed{6}$

$c_3 = (3)^3 - 3 = 27 - 3 = \boxed{24}$

$c_4 = (4)^3 - 4 = 64 - 4 = \boxed{60}$

$c_5 = (5)^3 - 5 = 125 - 5 = \boxed{120}$

$c_6 = (6)^3 - 6 = 216 - 6 = \boxed{210}$

100th term:

$c_{100} = (100)^3 - 100 = 1{,}000{,}000 - 100 = \boxed{999{,}900}$

⑤ $a_1 = 8$ and $a_n = a_{n-1} - 4,\ n \geq 2$

First four terms:

$a_1 = \boxed{8}$ Given

$a_2 = a_{2-1} - 4 = a_1 - 4 = 8 - 4 = \boxed{4}$

$a_3 = a_{3-1} - 4 = a_2 - 4 = 4 - 4 = \boxed{0}$

$a_4 = a_{4-1} - 4 = a_3 - 4 = 0 - 4 = \boxed{-4}$

Do you see a pattern? 8th term:

$8 \overset{-4}{,} 4 \overset{-4}{,} 0 \overset{-4}{,} -4,\ \underline{-8},\ \underline{-12},\ \underline{-16},\ \underline{-20}$

$\therefore, \boxed{a_8 = -20}$

⑦ $b_1 = 2$ and $b_{k+1} = 3b_k$, for $k \geq 1$

First four terms:

$b_1 = \boxed{2}$ Given

$b_{1+1} = b_2 = 3b_1 = 3(2) = \boxed{6}$

$b_{2+1} = b_3 = 3b_2 = 3(6) = \boxed{18}$

$b_{3+1} = b_4 = 3b_3 = 3(18) = \boxed{54}$

8th term:

$2 \overset{\times 3}{,} 6 \overset{\times 3}{,} 18 \overset{\times 3}{,} 54,\ \underline{\overset{162}{3(54)}},\ \underline{\overset{486}{3(162)}},\ \underline{\overset{1458}{3(486)}},\ \underline{3(1458)}$

$\boxed{\therefore,\ a_8 = 3(1458) = 4{,}374}$

⑨ $c_1 = 2$, $c_2 = -1$ and $c_{k+2} = c_k + c_{k+1}$, $k \geq 1$

First eight terms:

$c_1 = \boxed{2}$ } Given

$c_2 = \boxed{-1}$

$c_{1+2} = c_3 = c_1 + c_{1+1} = c_1 + c_2 = 2 + (-1) = \boxed{1}$

$c_{2+2} = c_4 = c_2 + c_{2+1} = c_2 + c_3 = -1 + 1 = \boxed{0}$

$c_{3+2} = c_5 = c_3 + c_{3+1} = c_3 + c_4 = 1 + 0 = \boxed{1}$

$c_{4+2} = c_6 = c_4 + c_{4+1} = c_4 + c_5 = 0 + 1 = \boxed{1}$

$c_{5+2} = c_7 = c_5 + c_{5+1} = c_5 + c_6 = 1 + 1 = \boxed{2}$

$c_{6+2} = c_8 = c_6 + c_{6+1} = c_6 + c_7 = 1 + 2 = \boxed{3}$

㉑ $\overset{a_1}{6}, 10 \overset{+4}{,} 14 \overset{+4}{,} 18, \ldots$

a) Common difference: $\boxed{4 = d}$

b) Tenth term: $a_n = a_1 + (n-1)d$

$a_{10} = 6 + (10-1)(4)$

$a_{10} = 6 + 9(4)$

$a_{10} = 6 + 36$

$\boxed{a_{10} = 42}$

c) Recursive formula (rule):

$a_n = a_{n-1} + d$, for $n \geq 2$

$\boxed{\therefore,\ a_n = a_{n-1} + 4, \text{ for } n \geq 2 \text{ and } a_1 = 6}$

d) Explicit formula (rule):

$a_n = a_1 + (n-1)d$ or $a_n = a_1 + d(n-1)$

$a_n = 6 + (n-1)(4)$

$a_n = \underline{\underline{6}} + 4n \underline{\underline{-4}}$

$\boxed{a_n = 2 + 4n}$ or $\boxed{a_n = 4n + 2}$

(23) $\overset{a_1}{-5}, \overset{+3}{-2}, \overset{+3}{1}, \overset{+3}{4}, \ldots$

a) Common difference: $\boxed{3 = d}$

b) Tenth term:

$a_n = a_1 + (n-1)d$

$a_{10} = -5 + (10-1)(3)$

$a_{10} = -5 + (9)(3)$

$a_{10} = -5 + 27$

$\boxed{a_{10} = 22}$

c) Recursive rule:

$\boxed{a_n = a_{n-1} + 3, \text{ for } n \geq 2 \text{ and } a_1 = -5}$

d) Explicit rule:

$a_n = a_1 + (n-1)d$

$a_n = -5 + (n-1)(3)$

$a_n = -5 + 3n - 3$

$\boxed{a_n = -8 + 3n}$ or $\boxed{a_n = 3n - 8}$

(29) Given: $a_4 = -8$, $a_7 = 4$

Need: d, a_1, and recursive rule

$\underset{}{\overset{?}{\_\_}}, \_\_, \_\_, \underset{a_4}{-8}, \_\_, \_\_, \underset{a_7}{4}$

Use explicit rule to set up problem: $a_n = a_1 + (n-1)d$

$a_7 = a_1 + (7-1)d$ $\qquad$ $a_4 = a_1 + (4-1)d$

$a_7 = a_1 + 6d$ $\qquad$ $a_4 = a_1 + 3d$

$$\underbrace{① \; 4 = a_1 + 6d \qquad ② \; -8 = a_1 + 3d}$$

System of equations ☺
Solve for d and a_1

$$\left.\begin{array}{ll} ① & 4 = a_1 + 6d \\ ② & -1(-8 = a_1 + 3d) \end{array}\right\} \Rightarrow \begin{array}{l} 4 = \cancel{a_1} + 6d \\ 8 = \cancel{-a_1} - 3d \\ \hline \frac{12}{3} = \frac{3d}{3} \\ \boxed{4 = d} \end{array}$$

Substitute 4 in for d in any of the two equations above:

$4 = a_1 + 6d$

$4 = a_1 + 6(4)$

$$\begin{array}{l} 4 = a_1 + \cancel{24} \\ -24 \quad\;\; \cancel{-24} \\ \hline \boxed{-20 = a_1} \end{array}$$

$\boxed{\therefore \text{ Recursive rule: } a_n = a_{n-1} + 4, \text{ for } n \geq 2,\ a_1 = -20}$

PA 7-6 p. 656 #2-10 even, 25, 27, 31

② $V_n = \frac{4}{n+2}$

First six terms:

$V_1 = \frac{4}{1+2} = \boxed{\frac{4}{3}}$

$V_2 = \frac{4}{2+2} = \frac{4}{4} = \boxed{1}$

$V_3 = \frac{4}{3+2} = \boxed{\frac{4}{5}}$

$V_4 = \frac{4}{4+2} = \frac{4}{6} = \boxed{\frac{2}{3}}$

$V_5 = \frac{4}{5+2} = \boxed{\frac{4}{7}}$

$V_6 = \frac{6}{6+2} = \frac{4}{8} = \boxed{\frac{1}{2}}$

100th term:

$V_{100} = \frac{4}{100+2} = \frac{4}{102} = \boxed{\frac{2}{51}}$

④ $d_n = n^2 - 5n$

First six terms:

$d_1 = (1)^2 - 5(1) = 1 - 5 = \boxed{-4}$

$d_2 = (2)^2 - 5(2) = 4 - 10 = \boxed{-6}$

$d_3 = (3)^2 - 5(3) = 9 - 15 = \boxed{-6}$

$d_4 = (4)^2 - 5(4) = 16 - 20 = \boxed{-4}$

$d_5 = (5)^2 - 5(5) = 25 - 25 = \boxed{0}$

$d_6 = (6)^2 - 5(6) = 36 - 30 = \boxed{6}$

100th term:

$d_{100} = (100)^2 - 5(100)$
$= 10{,}000 - 500$
$\boxed{d_{100} = 9{,}500}$

⑥ $u_1 = -3$, $u_{k+1} = u_k + 10$, for $k \ge 1$

First four terms:

$u_1 = \boxed{-3}$ Given

$u_{1+1} = u_2 = u_1 + 10 = -3 + 10 = \boxed{7}$

$u_{2+1} = u_3 = u_2 + 10 = 7 + 10 = \boxed{17}$

$u_{3+1} = u_4 = u_3 + 10 = 17 + 10 = \boxed{27}$

Pattern:

$-3, \overset{10}{7}, \overset{10}{17}, \overset{10}{27}, \overset{10}{\underline{37}}, \underline{47}, \underline{57}, \underline{67}$

8th term:

$\boxed{\therefore,\ a_8 = 67}$

⑧ $V_1 = 0.75$, $V_n = (-2)V_{n-1}$, for $n \ge 2$.

First four terms:

$V_1 = 0.75$ Given

$V_2 = (-2)V_{2-1} = (-2)V_1 = (-2)(0.75) = \boxed{-1.5}$

$V_3 = (-2)V_{3-1} = (-2)V_2 = (-2)(-1.5) = \boxed{3}$

$V_3 = (-2)V_{4-1} = (-2)V_3 = (-2)(3) = \boxed{-6}$

Pattern:

$0.75, \overset{\times(-2)}{-1.5}, \overset{\times(-2)}{3}, \overset{\times(-2)}{-6}, \overset{12}{\underline{(-2)(-6)}}, \overset{-24}{\underline{-2(12)}}, \overset{48}{\underline{(-2)(-24)}}, \overset{-96}{\underline{(-2)(48)}}$

8th term:

$\boxed{\therefore,\ a_8 = -96}$

⑩ $c_1 = -2$, $c_2 = 3$, $c_k = c_{k-2} + c_{k-1}$, $k \ge 3$

$c_1 = \boxed{-2}$

$c_2 = \boxed{3}$

$c_3 = c_{3-2} + c_{3-1} = c_1 + c_2 = -2 + 3 = \boxed{1}$

$c_4 = c_{4-2} + c_{4-1} = c_2 + c_3 = 3 + 1 = \boxed{4}$

$c_5 = c_{5-2} + c_{5-1} = c_3 + c_4 = 1 + 4 = \boxed{5}$

$c_6 = c_{6-2} + c_{6-1} = c_4 + c_5 = 4 + 5 = \boxed{9}$

$c_7 = c_{7-2} + c_{7-1} = c_5 + c_6 = 5 + 9 = \boxed{14}$

$c_8 = c_{8-2} + c_{8-1} = c_6 + c_7 = 9 + 14 = \boxed{23}$

㉕ $\overset{a_1}{2}, 6, 18, 54, \ldots$

(a) Common ratio: $\boxed{3 = r}$

$\frac{6}{2} = 3$, $\frac{18}{6} = 3$, $\frac{54}{18} = 3$ ✓

(b) 8th term:

$a_n = a_1 r^{n-1}$

$a_8 = (2)(3)^{8-1}$

$a_8 = 2 \cdot 3^7$

$a_8 = 2(2187)$

$\boxed{a_8 = 4{,}374}$

(c) Recursive rule:

$a_n = a_{n-1} \cdot r$, for $n \ge 2$

$\boxed{a_n = a_{n-1} \cdot 3, \text{ for } n \ge 2,\ a_1 = 2}$

or

$\boxed{a_n = 3a_{n-1}, \text{ for } n \ge 2,\ a_1 = 2}$

d) Explicit rule:

$a_n = a_1 \cdot r^{n-1}$

$\boxed{a_n = 2(3)^{n-1}}$

or

$a_n = 2(3)^n(3)^{-1}$

$a_n = 2(3)^n \cdot \frac{1}{3^1}$

$a_n = 2(3)^n \cdot \frac{1}{3}$

$\therefore, \boxed{a_n = \frac{2}{3}(3)^n}$

(27) $\overset{=a_1}{1}, -2, 4, -8, 16, \ldots$

(a) Common ratio: $\boxed{-2=r}$

$\frac{-2}{1}=-2$, $\frac{4}{-2}=-2$, $\frac{-8}{4}=-2$ ✓

(b) 8th term:

$a_n=a_1r^{n-1}$

$a_8=1(-2)^{8-1}$

$a_8=1(-2)^7$

$\boxed{a_8=-128}$

(c) Recursive rule:

$a_n=a_{n-1}\cdot r$, for $n\geq 2$

$\boxed{a_n=a_{n-1}(-2), \text{ for } n\geq 2, a_1=1}$

or

$\boxed{a_n=-2\cdot a_{n-1}, \text{ for } n\geq 2, a_1=1}$

(d) Explicit rule:

$a_n=a_1\cdot r^{n-1}$

$a_n=1\cdot(-2)^{n-1}$

$\boxed{a_n=(-2)^{n-1}}$

or

$a_n=(-2)^n(-2)^{-1}$

$a_n=(-2)^n\left(-\frac{1}{2}\right)$

$\boxed{a_n=\left(-\frac{1}{2}\right)(-2)^n}$

(31) Given: $a_2=3, a_8=192$

Need: a_1, common ratio, and explicit rule for n^{th} term

Use explicit rule to set up problem: $a_n=a_1\cdot r^{n-1}$

$a_8=a_1r^{8-1}$ $\qquad a_2=a_1r^{2-1}$

Equation ① $192=a_1r^7$ $\xleftarrow{\text{sub in}}$ $3=a_1r$ Equation ②

$192=a_1r\cdot r^6$

$192=3\cdot r^6$

$\frac{192}{3}=\frac{3r^6}{3}$

$64=r^6$

$\pm\sqrt[6]{64}=\sqrt[6]{r^6}$

$\pm\sqrt[6]{2^6}=r$

$\boxed{\pm 2=r}$

$\therefore$, $\underset{a_1}{\frac{3}{2}}$, 3, 6, 12, 24, 48, 96, $\underset{a_8}{192}$ (×2, ×2, ×2, ×2 …)

or

$\underset{a_1}{-\frac{3}{2}}$, 3, −6, 12, −24, 48, −96, $\underset{a_8}{192}$ (×(−2), ×(−2), ×(−2) …)

Visual of both sequences

sub in r into equation ① or ②

$3=a_1\cdot r$

$3=a_1(\pm 2)$

$\frac{3}{\pm 2}=a_1$

$\boxed{\therefore, a_1=\frac{3}{2} \text{ or } a_1=-\frac{3}{2}}$

Explicit rule:

$\boxed{a_n=\left(\pm\frac{3}{2}\right)(\pm 2)^{n-1}}$

$\boxed{a_n=\frac{3}{2}(2)^{n-1}}$

$a_n=3\cdot\frac{1}{2}(2)^{n-1}$

Law of negative exponents

$a_n=3\cdot 2^{-1}(2)^{n-1}$

$a_n=3(2)^{-1+n-1}$

$\boxed{a_n=3(2)^{n-2}}$

or

$\boxed{a_n=-\frac{3}{2}(-2)^{n-1}}$

$a_n=3\left(-\frac{1}{2}\right)(-2)^{n-1}$

$a_n=3(-2)^{-1}(-2)^{n-1}$

$a_n=3(-2)^{-1+n-1}$

$\boxed{a_n=3(-2)^{n-2}}$

PA 7-7 p.657 # 43-45 all; p.664 # 1-11 odd

(43) First two terms of geo. seq. negative. Is the third? Justify.

$\boxed{\text{TRUE.}}$ Take, for example, the following sequence: $-2, -8, \dots$

Common ratio: $r = \frac{-8}{-2} = 4$, which is positive. To get the third term we multiply $4(-8) = -32$. Notice $a_3 = -32$, a negative term.

(44) First two terms of arith. seq. are positive. Is the third? Justify.

$\boxed{\text{FALSE.}}$ Take, for example, the following seq: $3, 1, -1, -3, -5, \dots$ (-2, -2, -2, -2)

Notice common diff: $d = -2$

So $a_3 = -1$, NOT a positive term.

(45) $a_1 = 2$, $a_2 = 8$

$\therefore, d = 8 - 2 = 6$

$\therefore, a_4 = 2 + (4-1)(6)$

$a_4 = 2 + (3)(6)$

$a_4 = 2 + 18$

$\boxed{\therefore, a_4 = 20, (A)}$

(1) $\underbrace{-7}_{a_1} \overset{+6}{-} 1 \overset{+6}{+} 5 + \cdots + \underbrace{53}_{a_n = 11}$ → finite arithmetic series

"1st term" "nth term" "11 terms"

Need to figure out # of terms in sequence, so need n. of course this is easy here b/c we can follow pattern up to 53 but what if an was in the millions?

$a_1 = -7$, $a_n = 53$, $d = 6$

$a_n = a_1 + (n-1)d$

$53 = -7 + (n-1)(6)$

$53 = -7 + 6n - 6$

$53 = -13 + 6n$

$+13 \quad +13$

$\frac{66}{6} = \frac{6n}{6}$

$\boxed{11 = n}$

"11 terms"

Explicit rule:

$a_n = -7 + (n-1)(6)$

$a_n = -7 + 6n - 6$

$\boxed{a_n = 6n - 13}$

Summation Notation:

$\boxed{\sum_{k=1}^{11} 6k - 13}$

(3) $1 + 4 + 9 + \cdots + (n+1)^2$

$1^2 + 2^2 + 3^2 + \cdots + (n+1)^2$

Summation Notation:

$\boxed{\sum_{k=1}^{n+1} k^2}$

(5) $6 - 12 + 24 - 48 + \cdots$ → Infinite # of terms

This is a geometric series (infinite): $r = \frac{-48}{24} = \frac{24}{-12} = -\frac{12}{6} = -2$

Explicit rule:

$\boxed{a_n = 6(-2)^{n-1}}$ or

$a_n = 6(-2)^{n-1}$

$a_n = 6(-2)^n(-2)^{-1}$

$a_n = 6(-2)^n(-\frac{1}{2})$

$a_n = 6(-\frac{1}{2})(-2)^n$

$\boxed{a_n = -3(-2)^n}$

Summation Notation:

$\boxed{\sum_{k=1}^{\infty} 6(-2)^{k-1}}$ or

$\boxed{\sum_{k=1}^{\infty} -3(-2)^k}$ or

$\boxed{\sum_{k=0}^{\infty} 6(-2)^k}$

(7) $\underset{a_1}{-7}, \overset{+4}{-3}, \overset{+4}{1}, 5, 9, \underset{a_n}{13}$ → Six terms. Easy!

Arithmetic sequence ✓

Sum formula:

$\sum_{k=1}^{n} a_k = \frac{n}{2}(a_1 + a_n)$

$\therefore, \sum_{k=1}^{6} a_k = \frac{6}{2}(-7 + 13)$

$= 3(6)$

$= \boxed{18}$

$\boxed{\therefore, \text{The sum of sequence is } 18.}$

(9) $\underset{a_1}{1}, 2, 3, 4, \dots, \overset{a_n}{80}$ → 80 terms. Easy!

Arithmetic Sequence ✓

Sum formula:

$\sum_{k=1}^{80} a_k = \frac{80}{2}(1 + 80)$

$= 40(81)$

$= \boxed{3,240}$

$\boxed{\therefore, \text{The sum of seq is } 3,240}$

(11) $\overset{a_1}{117}, \overset{-7}{110}, \overset{-7}{103}, \dots, \overset{a_n}{33}$

How many terms? Use explicit formula to find n.

$a_n = a_1 + (n-1)d$

$33 = 117 + (n-1)(-7)$

$33 = 117 - 7n + 7$

$33 = 124 - 7n$

$-124 \quad -124$

$\frac{-91}{-7} = \frac{-7n}{-7}$

$\boxed{13 = n}$ "13 terms"

$\sum_{k=1}^{13} a_k = \frac{13}{2}(117 + 33) = \frac{13}{2}(150) = 13(75) = \boxed{975}$

$\boxed{\therefore, \text{The sum of seq. is } 975}$

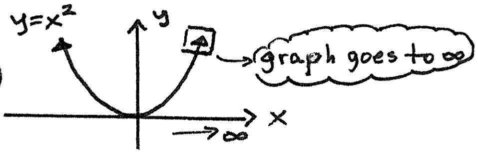
PA 7-8 p. 656 # 11, 13 ; p. 664 # 13-29 odd

(11) $1, 4, 9, 16, \ldots, n^2, \ldots$

$1^2, 2^2, 3^2, 4^2, \ldots, n^2, \ldots$

Notice $\lim_{n\to\infty} n^2 = \infty$ b/c

Recall end behavior of a function ☺



$\boxed{\therefore\text{, sequence diverges}}$

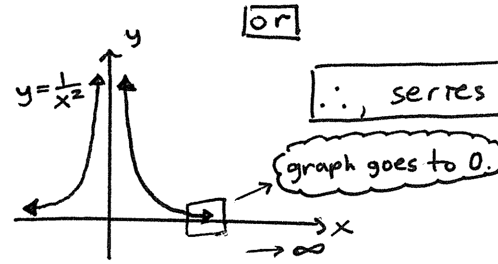
(13) $\frac{1}{1}, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}, \ldots$

$\frac{1}{1^2}, \frac{1}{2^2}, \frac{1}{3^2}, \frac{1}{4^2}, \ldots, \frac{1}{n^2}, \ldots$

Notice $\lim_{n\to\infty} \frac{1}{n^2} = 0$ b/c

degree of num. < degree of denominator

$\boxed{\text{or}}$



$\boxed{\therefore\text{, series converges to } 0.}$

(13) $\underbrace{3}_{a_1}, 6, 12, \ldots, \underbrace{12,288}_{a_n}$

$r = \frac{6}{3} = \frac{12}{6} = 2$

$a_n = a_1 r^{n-1}$

$\frac{12,288}{3} = \frac{\cancel{3}(2)^{n-1}}{\cancel{3}}$

$4096 = 2^{n-1}$

$2^{12} = 2^{n-1}$

$12 = n-1$

$+1 \quad +1$

$\boxed{13=n}$

Sum of geo. seq formula:

$$\sum_{k=1}^{n} a_k = \frac{a_1(1-r^n)}{1-r} \quad \text{OR} \quad \sum_{k=1}^{n} a_1 \cdot r^{k-1} = \frac{a_1(1-r^n)}{1-r}$$

$$\therefore, \sum_{k=1}^{13} 3(2)^{k-1} = \frac{3(1-2^{13})}{1-2} = \frac{3(1-2^{13})}{-1} = -3(1-2^{13}) = \boxed{24,573}$$

$\boxed{\therefore\text{, the geo. seq. sum is } 24,573}$

(15) $\underbrace{42}_{a_1}, 7, \frac{7}{6}, \ldots, \underbrace{42\left(\frac{1}{6}\right)^8}_{a_n}$

$r = \frac{7}{42} = \frac{1}{6}$

$r = \frac{\frac{7}{6}}{7} = \frac{7}{6} \cdot \frac{1}{7} = \frac{1}{6}$

$a_n = a_1 r^{n-1}$

$42\left(\frac{1}{6}\right)^8 = 42\left(\frac{1}{6}\right)^{n-1}$

$8 = n-1$

$\boxed{9=n}$

$$\sum_{k=1}^{9} 42\left(\tfrac{1}{6}\right)^{k-1} = \frac{42\left(1-\left(\frac{1}{6}\right)^9\right)}{1-\frac{1}{6}}$$

$$= \frac{42\left(1-(6^{-1})^9\right)}{1-\frac{1}{6}}$$

$$= \frac{42(1-6^{-9})}{\frac{5}{6}}$$

$$= 42 \cdot \tfrac{6}{5}(1-6^{-9}) = \boxed{50.4}$$

$\boxed{\therefore\text{, The sum of geo. seq is } \approx 50.4}$

(17) $2 \overset{+3}{,} 5 \overset{+3}{,} 8, \ldots, a_{10=n}$

This is Arithmetic Sequence

$a_1 = 2$, $n = 10$, $d = 3$ Use $\sum_{k=1}^{n} a_k = \frac{n}{2}(2a_1 + (n-1)d)$

$$\sum_{k=1}^{10} a_k = \tfrac{10}{2}\left(2\cdot 2 + (10-1)(3)\right)$$

$$= 5(4 + 9\cdot 3)$$

$$= 5(4+27)$$

$$= 5(31)$$

$$= \boxed{155}$$

$\boxed{\therefore\text{, The sum of arith seq is } 155.}$

(19) $\overset{a_1}{4}, -2, 1, -\frac{1}{2}, \ldots \; a_{12=n}$ Geometric sequence

$r = -\frac{2}{4} = -\frac{1}{2}$

$$\sum_{k=1}^{12} 4\left(-\tfrac{1}{2}\right)^{k-1} = \frac{4\left(1-\left(-\frac{1}{2}\right)^{12}\right)}{1-\left(-\frac{1}{2}\right)}$$

$1 + \frac{1}{2} = \frac{2}{2} + \frac{1}{2} = \frac{3}{2}$

$$= \frac{4\left(1-(2^{-1})^{12}\right)}{\frac{3}{2}}$$

$\left(-\frac{1}{2}\right)^{12} = \left(\frac{1}{2}\right)^{12}$

$\frac{1}{2^1} = 2^{-1}$

$$= 4 \cdot \tfrac{2}{3}(1-2^{-12})$$

$$= \boxed{2.666}$$

$\boxed{\therefore\text{, the sum of geo. seq is } \approx 2.666}$

(21) $-1, 11, -121, \ldots, a_{9=n}$

$r = \frac{11}{-1} = -11$ $\quad n=9$ $\quad$ Geometric seq.

$r = -\frac{121}{11} = -11$

$$\sum_{k=1}^{9} (-1)(-11)^{k-1} = \frac{-1(1-(-11)^9)}{1-(-11)}$$

$$= \frac{-1(1-(-11^9))}{1+11}$$

$$= \frac{-1(1+11^9)}{12}$$

$$= \boxed{-196,495,641}$$

$\therefore$, the sum of geo. seq is $-196,495,641$

(25) $\underbrace{6}_{a_1} + 3 + \frac{3}{2} + \frac{3}{4} + (\cdots) \to$ Infinite # of terms

$r = \frac{3}{6} = \frac{1}{2}$

$r = \frac{\frac{3}{2}}{3} = \frac{3}{2} \cdot \frac{1}{3} = \frac{1}{2}$ $\Big\}$ Infinite geometric

$\sum_{k=1}^{n} a_k = \frac{a_1}{1-r}$ OR $\sum_{k=1}^{n} a_1 \cdot r^{k-1} = \frac{a_1}{1-r}$

check: $|r| < 1$

$|\frac{1}{2}| < 1$

$\frac{1}{2} < 1$ ✓ converges

$$\therefore, \sum_{k=1}^{\infty} 6\left(\tfrac{1}{2}\right)^{k-1} = \frac{6}{1-\frac{1}{2}}$$

$$= \frac{6}{\frac{1}{2}}$$

$$= 6 \cdot \frac{2}{1}$$

$$= 6 \cdot 2$$

$$= \boxed{12}$$

$\therefore$, the series converges to 12

(27) $\underbrace{\frac{1}{64}}_{a_1} + \frac{1}{32} + \frac{1}{16} + \frac{1}{8} + (\cdots) \to$ Infinite # of terms

$r = \frac{\frac{1}{32}}{\frac{1}{64}} = \frac{1}{32} \cdot \frac{64}{1} = \frac{64}{32} = 2$

But $|r| < 1$

$|2| \nless 1$

$2 \nless 1$ ☹

$\therefore$, The series doesn't have a finite limit.

$\therefore$, The series diverges.

(23) (a) $0.3 + 0.03 + 0.003 + 0.0003 + \cdots$

Partial sums:

$S_1 = 0.3$

$S_2 = 0.3 + 0.03 = 0.33$

$S_3 = 0.3 + 0.03 + 0.003 = 0.333$

$S_4 = 0.3 + 0.03 + 0.003 + 0.0003 = 0.3333$

$S_5 = 0.3 + 0.03 + 0.003 + 0.0003 + 0.00003 = 0.33333$

$S_6 = 0.3 + 0.03 + 0.003 + 0.0003 + 0.00003 + 0.000003 = 0.333333$

$\therefore$, Partial sums appear to approch $0.\overline{3} = \frac{1}{3}$.

$\therefore$, Infinite sums have a finite limit of $\frac{1}{3}$, "convergent"

(b) $1 - 2 + 3 - 4 + 5 - 6 + \cdots$

Partial sums:

$S_1 = 1$

$S_2 = 1 - 2 = -1$

$S_3 = 1 - 2 + 3 = 2$

$S = 1 - 2 + 3 - 4 = -2$

$S_5 = 1 - 2 + 3 - 4 + 5 = 3$

$S_6 = 1 - 2 + 3 - 4 + 5 - 6 = -3$

$\therefore$, Partial sums DO NOT appear to reach a finite limit.

$\therefore$, Infinite sum is "divergent."

(29) $\sum_{j=1}^{\infty} 3\left(\frac{1}{4}\right)^j$ Infinite geometric series

Write out the terms:

$$\sum_{j=1}^{\infty} 3\left(\tfrac{1}{4}\right)^j = 3\left(\tfrac{1}{4}\right)^1 + 3\left(\tfrac{1}{4}\right)^2 + 3\left(\tfrac{1}{4}\right)^3 + 3\left(\tfrac{1}{4}\right)^4 + \cdots$$

$$= \underbrace{\frac{3}{4}}_{a_1} + \frac{3}{16} + \frac{3}{64} + \frac{3}{256} + \cdots$$

$r = \frac{3}{16} / \frac{3}{4} = \frac{3}{16} \cdot \frac{4}{3} = \frac{4}{16} = \frac{1}{4}$ ✓

$|r| < 1$

$|\frac{1}{4}| < 1$

$\frac{1}{4} < 1$ ✓

$$\sum_{j=1}^{\infty} 3\left(\tfrac{1}{4}\right)^j = \frac{\frac{3}{4}}{1-\frac{1}{4}}$$

$$= \frac{\frac{3}{4}}{\frac{3}{4}}$$

$$= \boxed{1}$$

$\therefore$, the inf. geo. series converges and its sum is 1.

PA 7-9 p. 656 # 37, 39; p. 664 # 31-39 odd

(37) 1 meter = 100 cm

Bungy-bungy tree grows an avg. of 2.3 cm per week

Sequence of weekly height over 1 year (52 weeks in one year)

$a_1 = 7\text{ meters} = 700\text{ cm}$
$a_2 = 700 + 2.3 = 702.3\text{ cm}$
$a_3 = 702.3 + 2.3 = 704.6\text{ cm}$
$a_4 = 704.6 + 2.3 = 706.9\text{ cm}$

$a_1 = 700\text{ cm}$
$d = 2.3$ (common difference)

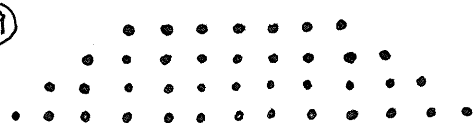
$a_n = a_1 + (n-1)d$
$a_n = 700 + (n-1)(2.3)$
$a_n = 700 + 2.3n - 2.3$
$a_n = 2.3n + 697.7$

We also need last two terms:

$\boxed{a_{51} = 2.3(51) + 697.7 = 815}$

$\boxed{a_{52} = 2.3(52) + 697.7 = 817.3}$

(39)



25 rows in total

$a_1 = 7$ seats, $d = 2$, $n = 25$ (# of rows/terms)

How many seats?

$$\sum_{k=1}^{25} a_k = \frac{n}{2}(2a_1 + (n-1)d)$$
$$= \frac{25}{2}(2\cdot 7 + (25-1)(2))$$
$$= \frac{25}{2}(14 + 24\cdot 2)$$
$$= \frac{25}{2}(14+48)$$
$$= \frac{25}{2}(62)$$
$$= 25(31)$$
$$= \boxed{775}$$

$\therefore$, There are 775 seats in section J.

(31) $7.14141414\ldots = 7.\overline{14}$

$$7 + (\underbrace{0.14}_{a_1} + 0.0014 + 0.000014 + \cdots)$$

Infinite series

$$r = \frac{0.0014}{0.14} = 0.01$$

$|r| < 1$

$|0.01| < 1$

$0.01 < 1 \Rightarrow$ series converges

$$\therefore \sum_{K=1}^{\infty} a_K = \frac{a_1}{1-r} = \frac{0.14}{1-0.01} = \frac{0.14}{0.99} = \boxed{\frac{14}{99}}$$

$\therefore\ 7.14141414\ldots$

$$= 7 + (0.14 + 0.0014 + 0.000014 + \cdots)$$

$$= 7 + \frac{14}{99}$$

$$= \frac{7}{1}\cdot\frac{99}{99} + \frac{14}{99}$$ "common den."

$$= \frac{693}{99} + \frac{14}{99}$$

$$= \frac{693+14}{99}$$

$$= \boxed{\frac{707}{99}}$$

(33) $-17.268268268\ldots = -17.\overline{268}$

$$-17 - (\underbrace{0.268}_{a_1} + 0.000268 + 0.000000268 + \cdots)$$

Infinite series

$$r = \frac{0.000268}{0.268} = 0.001$$

$|r| < 1$

$|0.001| < 1 \Rightarrow$ series converges

$$\therefore \sum_{K=1}^{\infty} a_K = \frac{a_1}{1-r} = \frac{0.268}{1-0.001} = \frac{0.268}{0.999} = \boxed{\frac{268}{999}}$$

$\therefore\ -17.268268268\ldots$

$$= -17 - (0.268 + 0.000268 + \cdots)$$

$$= -17 - \frac{268}{999}$$

$$= -\frac{17}{1}\cdot\frac{999}{999} - \frac{268}{999}$$

$$= \frac{-16{,}983}{999} - \frac{268}{999}$$

$$= \frac{-16{,}983 - 268}{999}$$

$$= \boxed{-\frac{17{,}251}{999}}$$

(35) (a) $r = \frac{22{,}000}{20{,}000} = \frac{22}{20} = \frac{11}{10} = \boxed{1.1}$

(b) $\boxed{b_n = 20{,}000(1.1)^n, \text{ where } b_0 = 20{,}000}$

(c) Sum of balances from 1996 to 2006

$$\sum_{K=1}^{11} 20{,}000(1.1)^{K-1} = \frac{20{,}000(1-1.1^{11})}{1-1.1} = \boxed{\$370{,}623.34}$$

(37) $$\underbrace{120\left(1+\frac{0.07}{12}\right)^0}_{a_1} + \underbrace{120\overbrace{\left(1+\frac{0.07}{12}\right)}^{r}{}^1}_{a_2} + \cdots + \underbrace{120\left(1+\frac{0.07}{12}\right)^{119}}_{a_{120}}$$

(a) First term: $a_1 = 120\left(1+\frac{0.07}{12}\right)^0 = 120(1) = \boxed{120}$

Common ratio: $r = \dfrac{120\left(1+\frac{0.07}{12}\right)^1}{120\left(1+\frac{0.07}{12}\right)^0} = \dfrac{\cancel{120}\left(1+\frac{0.07}{12}\right)^1}{\cancel{120}} = \boxed{1+\frac{0.07}{12}}$

(b) Sum formula:

$$\sum_{k=1}^{120} a_k = \frac{120\left(1-\left(1+\frac{0.07}{12}\right)^{120}\right)}{1-\left(1+\frac{0.07}{12}\right)} = \boxed{\$\,20,770.18}$$

(39) * The ball is dropped from a height of 2 meters. (travels 2m down)

① Between the 1st and 2nd bounce the ball travels $2(0.9)$ m up and $2(0.9)$ m down. (the problem states rubber ball bounces 90% of height)

② Between the 2nd and 3rd bounce the ball travels $2(0.9)^2$ m up and $2(0.9)^2$ m down.

③ And so on...

∴, We can write this experiment as an infinite geometric series:

$$\underbrace{\left(2(0.9)+2(0.9)\right)}_{①} + \underbrace{\left(2(0.9)^2+2(0.9)^2\right)}_{②} + \underbrace{\left(2(0.9)^3+2(0.9)^3\right)}_{③} + \cdots$$

$= 4(0.9) + 4(0.9)^2 + 4(0.9)^3 + \cdots$

$= 4(\underbrace{0.9}_{a_1} + 0.9^2 + 0.9^3 + \ldots)$ (Factor 4) $\left\{r = \frac{0.9^2}{0.9} = 0.9\right.$, $\left\{|0.9|<1\right.$, convergent

$= 4\cdot\left(\frac{0.9}{1-0.9}\right)$ (Apply formula for inf. geo. series : $\frac{a_1}{1-r}$)

$= 4(9)$

$= 36$

∴, The ball travels 36m after the first bounce.

∴, Since the ball is dropped from a height of 2 meters, the total distance traveled by the ball is $36\text{m} + 2\text{m}^{*} = \boxed{38\text{m}}$:)