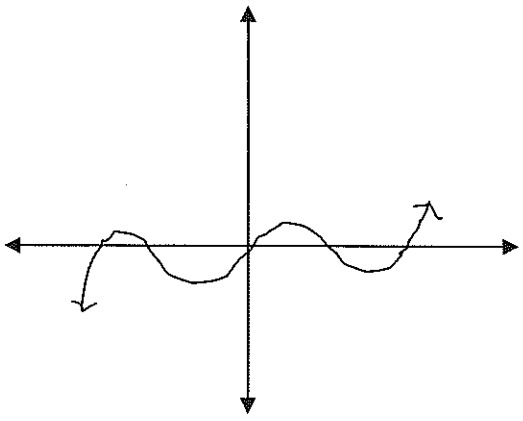
Advanced Algebra

Unit 7 Practice: Analyze Functions



As $x \to \infty$, $f(x) \to \infty$
As $x \to -\infty$, $f(x) \to -\infty$

Practice/Apply

Study Notebook

Have students—
add the definitions/examples of the vocabulary terms to their Vocabulary Builder worksheets for Chapter 7.
Copy the information in the Concept Summary about end behavior shown on p. 349.
Include any other item(s) that they find helpful in mastering the skills in this lesson.

About the Exercises...

Organization by Objective
Polynomial Functions: 16–38
Graphs of Polynomial Functions: 39–44

Odd/Even Assignments
Exercises 16–44 are structured so that students practice the same concepts whether they are assigned odd or even problems.

Assignment Guide

Basic: 17–45 odd, 49–52, 56–70
Average: 17–45 odd, 46–52, 56–70
Advanced: 16–44 even, 46–67 (optional: 68–70)

Answers

Sample answer: Even-degree polynomial functions with positive leading coefficients have graphs in which $f(x) \to +\infty$ as $x \to +\infty$ and as $x \to -\infty$. Odd-degree polynomial functions with positive leading coefficients have graphs in which $f(x) \to +\infty$ as $x \to +\infty$ and $f(x) \to -\infty$ as $x \to -\infty$.

Check for Understanding

Concept Check

1. Explain why a constant polynomial such as $f(x) = 4$ has degree 0 and a linear polynomial such as $f(x) = x + 5$ has degree 1. **$4 = 4x^0$; $x = x^1$**
2. Describe the characteristics of the graphs of odd-degree and even-degree polynomial functions whose leading coefficients are positive. **See margin.**
3. **OPEN ENDED** Sketch the graph of an odd-degree polynomial function with a negative leading coefficient and three real roots. **See margin.**
4. Tell whether the following statement is *always*, *sometimes* or *never* true. Explain.
A polynomial function that has four real roots is a fourth-degree polynomial.

Sometimes; a polynomial function with 4 real roots may be a sixth-degree polynomial function with 2 imaginary roots. A polynomial function that has 4 real roots is *at least* a fourth-degree polynomial.

Guided Practice

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

5. $5x^6 - 8x^2$ **6; 5**
6. $2b + 4b^3 - 3b^5 - 7$ **5; −3**

Find $p(3)$ and $p(-1)$ for each function.

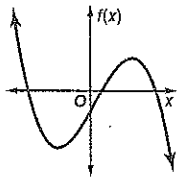
7. $p(x) = -x^3 + x^2 - x$ **−21; 3**
8. $p(x) = x^4 - 3x^3 + 2x^2 - 5x + 1$ **4; 12**

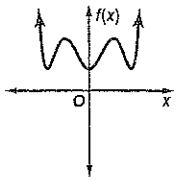
If $p(x) = 2x^3 + 6x - 12$ and $q(x) = 5x^2 + 4$, find each value.

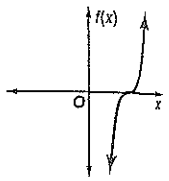
9. $p(a^3)$ **$2a^9 + 6a^3 - 12$**
10. $5[q(2a)]$ **$100a^2 + 20$**
11. $3p(a) - q(a + 1)$ **$6a^3 - 5a^2 + 8a - 45$**

GUIDED PRACTICE KEY	
Exercises	Examples
5, 6	1
7, 8, 15	2
9–11	3
12–14	4

For each graph,
a. describe the end behavior,
b. determine whether it represents an odd-degree or an even-degree polynomial function, and
c. state the number of real zeros.

12. 

13. 

14. 

12. a. $f(x) \to -\infty$ as $x \to +\infty$, $f(x) \to +\infty$ as $x \to -\infty$; b. odd; c. 3
13. a. $f(x) \to +\infty$ as $x \to +\infty$, $f(x) \to +\infty$ as $x \to -\infty$; b. even; c. 0
14. a. $f(x) \to +\infty$ as $x \to +\infty$, $f(x) \to -\infty$ as $x \to -\infty$; b. odd; c. 1

Application

15. **BIOLOGY** The intensity of light emitted by a firefly can be determined by $L(t) = 10 + 0.3t + 0.4t^2 - 0.01t^3$, where t is temperature in degrees Celsius and $L(t)$ is light intensity in lumens. If the temperature is 30°C, find the light intensity. **109 lumens**

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
16–21	1
22–29, 45	2
30–38	3
39–44, 46–48	4

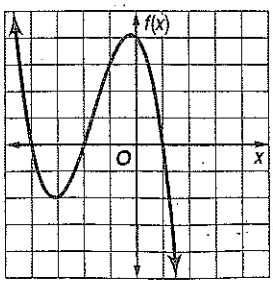
Extra Practice See page 842.

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

16. $7 - x$ **1; −1**
17. $(a + 1)(a^2 - 4)$ **3; 1**
18. $a^2 + 2ab + b^2$ **See margin.**
19. $6x^4 + 3x^2 + 4x - 8$ **4; 6**
20. $7 + 3x^2 - 5x^3 + 6x^2 - 2x$ **3; −5**
21. $c^2 + c - \frac{1}{c}$ **See margin.**

Find $p(4)$ and $p(-2)$ for each function.

22. $p(x) = 2 - x$ **−2; 4**
23. $p(x) = x^2 - 3x + 8$ **12; 18**
24. $p(x) = 2x^3 - x^2 + 5x - 7$ **125; −37**
25. $p(x) = x^5 - x^2$ **1008; −36**
26. $p(x) = x^4 - 7x^3 + 8x - 6$ **−166; 50**
27. $p(x) = 7x^2 - 9x + 10$ **86; 56**
28. $p(x) = \frac{1}{2}x^4 - 2x^2 + 4$ **100; 4**
29. $p(x) = \frac{1}{8}x^3 - \frac{1}{4}x^2 - \frac{1}{2}x + 5$ **7; 4**

3. Sample answer:


18. No, the polynomial contains two variables, a and b.
21. No, this is not a polynomial because the term $\frac{1}{c}$ cannot be written in the form x^n, where n is a nonnegative integer.

39a. $f(x) \to +\infty$ as $x \to +\infty$, $f(x) \to -\infty$ as $x \to -\infty$; b. odd; c. 3
40a. $f(x) \to +\infty$ as $x \to +\infty$, $f(x) \to +\infty$ as $x \to -\infty$; b. even; c. 4
41a. $f(x) \to -\infty$ as $x \to +\infty$, $f(x) \to -\infty$ as $x \to -\infty$; b. even; c. 0
42a. $f(x) \to +\infty$ as $x \to +\infty$, $f(x) \to -\infty$ as $x \to -\infty$; b. odd; c. 5
43a. $f(x) \to +\infty$ as $x \to +\infty$, $f(x) \to -\infty$ as $x \to -\infty$; b. odd; c. 1
44a. $f(x) \to -\infty$ as $x \to +\infty$, $f(x) \to -\infty$ as $x \to -\infty$; b. even; c. 2

32. $3a^4 - 2a^2 + 5$
34. $x^3 + 3x^2 + 4x + 3$
35. $3x^4 + 16x^2 + 26$

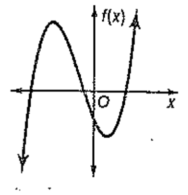
If $p(x) = 3x^2 - 2x + 5$ and $r(x) = x^3 + x + 1$, find each value.

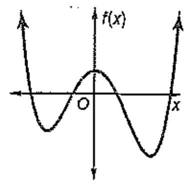
30. $r(3a)$ **$27a^3 + 3a + 1$**
31. $4p(a)$ **$12a^2 - 8a + 20$**
32. $p(a^2)$
33. $p(2a^3)$ **$12a^6 - 4a^3 + 5$**
34. $r(x + 1)$
35. $p(x^2 + 3)$
36. $2[p(x + 4)]$ **$6x^2 + 44x + 90$**
37. $r(x + 1) - r(x^2)$ **$-x^6 + x^3 + 2x^2 + 4x + 2$**
38. $3[p(x^2 - 1)] + 4p(x)$ **$9x^4 - 12x^2 - 8x + 50$**

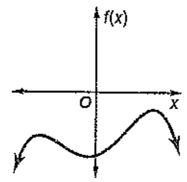
For each graph,

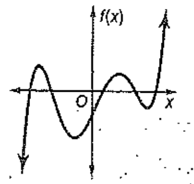
a. describe the end behavior,
b. determine whether it represents an odd-degree or an even-degree polynomial function, and
c. state the number of real zeros. **39–44. See margin.**

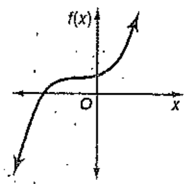
(x+1)(x-1)(x+2)(x-2)
$(x^2-1)(x^2-4)$ x^4-5x^2+4

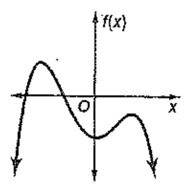
39.


40.


41.


42.


43.


44.


Theater

In 1997, *Cats* surpassed *A Chorus Line* as the longest-running Broadway show.

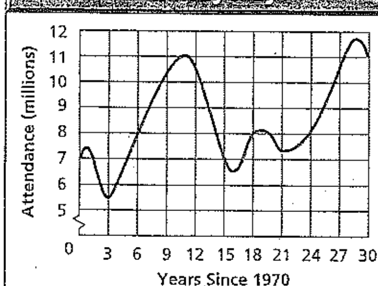
Source: www.newsherald.com

45. **ENERGY** The power generated by a windmill is a function of the speed of the wind. The approximate power is given by the function $P(s) = \frac{s^3}{1000}$, where s represents the speed of the wind in kilometers per hour. Find the units of power $P(s)$ generated by a windmill when the wind speed is 18 kilometers per hour. **5.832 units**

THEATER For Exercises 46–48, use the graph that models the attendance to Broadway plays (in millions) from 1970–2000.

46. Is the graph an odd-degree or even-degree function? **even**
47. Discuss the end behavior of the graph.
48. Do you think attendance at Broadway plays will increase or decrease after 2000? Explain your reasoning.

Broadway Plays



47. $f(x) \to -\infty$ as $x \to +\infty$; $f(x) \to -\infty$ as $x \to -\infty$

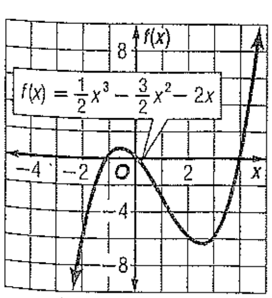
48. Sample answer: Decrease; the graph appears to be turning at $x = 30$ indicating a relative maximum at that point. So attendance will decrease after 2000.

CRITICAL THINKING For Exercises 49–52, use the following information.

The graph of the polynomial function $f(x) = ax(x - 4)(x + 1)$ goes through the point at (5, 15).

49. Find the value of a. $\frac{1}{2}$
50. For what value(s) of x will $f(x) = 0$? **−1, 0, 4**
51. Rewrite the function as a cubic function. $f(x) = \frac{1}{2}x^3 - \frac{3}{2}x^2 - 2x$
52. Sketch the graph of the function. **See margin.**

www.algebra2.com/self_check_quiz

52.


Study Guide and Intervention p. 375 (shown) and p. 376

Polynomial Functions

Polynomial in One Variable	A polynomial of degree n in one variable x is an expression of the form $a_0x^n + a_1x^{n-1} + \dots + a_{n-2}x^2 + a_{n-1}x + a_n$, where the coefficients $a_0, a_1, a_2, \dots, a_n$ represent real numbers, … zero, and n represents a nonnegative integer.

The degree of a polynomial in one variable is the greatest e… leading coefficient is the coefficient of the term with the high…

Polynomial Function	A polynomial function of degree n can be described by an equation of the form $P(x) = a_0x^n + a_1x^{n-1} + \dots + a_{n-2}x^2 + a_{n-1}x + a_n$, where the coefficients $a_0, a_1, a_2, \dots, a_n$ represent real numbers, a_0 is not zero, and n represents a nonnegative integer.

Example 1 What are the degree and leading coefficient of $3x^2 - 2$…

Rewrite the expression so the powers of x are in decreasing order.

$-2x^4 + x^3 + 3x^2 - 7$

This is a polynomial in one variable. The degree is 4, and the leading coeffici…

Example 2 Find $f(-5)$ if $f(x) = x^3 + 2x^2 - 10x + 20$.

$f(x) = x^3 + 2x^2 - 10x + 20$ Original function
$f(-5) = (-5)^3 + 2(-5)^2 - 10(-5) + 20$ Replace x with −5.
$= -125 + 50 + 50 + 20$ Evaluate.
$= -5$ Simplify.

Example 3 Find $g(a^2 - 1)$ if $g(x) = x^2 + 3x - 4$.

$g(x) = x^2 + 3x - 4$ Original function
$g(a^2 - 1) = (a^2 - 1)^2 + 3(a^2 - 1) - 4$ Replace x with $a^2 - 1$.
$= a^4 - 2a^2 + 1 + 3a^2 - 3 - 4$ Evaluate.
$= a^4 + a^2 - 6$ Simplify.

Exercises

State the degree and leading coefficient of each polynomial in one v… not a polynomial in one variable, explain why.

1. $3x^4 + 6x^3 - x^2 + 12$ 4; 3
2. $100 - 5x^3 + 10x^7$ 7; 10
3. $4x^6 + 6x^4 + \dots$
4. $4x^2 - 3xy + 16y^2$ not a polynomial in one variable; contains two variables
5. $8x^3 - 9x^5 + 4x^2 - 36$ 5; −9
6. $\frac{x^2}{18} - \frac{x^6}{25} + \dots$ 6; $-\frac{1}{25}$

Find $f(2)$ and $f(-5)$ for each function.

7. $f(x) = x^2 - 9$ −5; 16
8. $f(x) = 4x^3 - 3x^2 + 2x - 1$ 23; −586
9. $f(x) = 9x^3 - \dots$ 73; −1243

Skills Practice, p. 377 and Practice, p. 378 (shown)

State the degree and leading coefficient of each polynomial in one va… not a polynomial in one variable, explain why.

1. $(3x^2 + 1)(2x^2 - 9)$ 4; 6
2. $\frac{1}{5}a^3 - \frac{3}{5}a^2 + \frac{4}{5}a$ 3; $\frac{1}{5}$
3. $\frac{2}{m^2} + 3m - 12$ Not a polynomial; $\frac{2}{m^2}$ cannot be written in the form m^n for a nonnegative integer n.
4. $27 + 3xy^3 - 12x^2y^2 - 10y$ No, this polynomial con… variables, x and y.

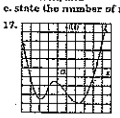
Find $p(-2)$ and $p(3)$ for each function.

5. $p(x) = x^3 - x^5$ 24; −216
6. $p(x) = -7x^2 + 5x + 9$ −29; −39
7. $p(x) = -x^3 \dots$ 0; −135
8. $p(x) = 3x^3 - x^2 + 2x - 5$ −37; 73
9. $p(x) = x^4 + \frac{1}{2}x^3 - \frac{1}{2}x$ 13; 93
10. $p(x) = \frac{1}{3}x^3 \dots$ −6; 24

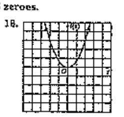
If $p(x) = 3x^2 - 4$ and $r(x) = 2x^2 - 5x + 1$, find each value.

11. $p(8a)$ $192a^2 - 4$
12. $r(a^2)$ $2a^4 - 5a^2 + 1$
13. … +
14. $r(x + 2)$ $2x^2 + 3x - 1$
15. $p(x^2 - 1)$ $3x^4 - 6x^2 - 1$
16. … + 2) … $x^2 + 6$

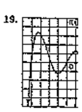
For each graph,
a. describe the end behavior,
b. determine whether it represents an odd-degree or an even-degree function, and
c. state the number of real zeroes.

17.


$f(x) \to +\infty$ as $x \to +\infty$, $f(x) \to +\infty$ as $x \to -\infty$; even; 2

18.


$f(x) \to +\infty$ as $x \to +\infty$, $f(x) \to +\infty$ as $x \to -\infty$; even; 1

19.


$f(x) \to +\infty$ … $f(x) \to -\infty$ … odd; 5

20. WIND CHILL The function $C(s) = 0.013s^2 - s - 7$ estimates the wind chi… $C(s)$ at 0°F for wind speeds s from 5 to 30 miles per hour. Estimate the wi… temperature at 0°F if the wind speed is 20 miles per hour. about −22°F

Reading to Learn Mathematics, p. 379

Pre-Activity Where are polynomial functions found in nature?

Read the introduction to Lesson 7-1 at the top of page 346 in…

- In the honeycomb cross section shown in your textbook, the… in the center, 6 hexagons in the second ring, and 12 hexag… ring. How many hexagons will there be in the fourth, fifth, … 18; 24; 30
- There is 1 hexagon in a honeycomb with 1 ring. There are… a honeycomb with 2 rings. How many hexagons are there i… with 3 rings, 4 rings, 5 rings, and 6 rings? 19; 37; 61; 91

Reading the Lesson

1. Give the degree and leading coefficient of each polynomial in one variable…

	degree	leading coefficient
a. $10x^3 + 3x^2 - x + 7$	3	10
b. $7y^2 - 2y^5 + y - 4y^3$	5	−2
c. 100	0	100

2. Match each description of a polynomial function from the list on the left w… corresponding end behavior from the list on the right.

a. even degree, negative leading coefficient iii
b. odd degree, positive leading coefficient iv
c. odd degree, negative leading coefficient ii
d. even degree, positive leading coefficient i

i. $f(x) \to +\infty$ as $x \to +\dots$ $f(x) \to +\infty$ as $x \to -\dots$
ii. $f(x) \to -\infty$ as $x \to +\dots$ $f(x) \to +\infty$ as $x \to -\dots$
iii. $f(x) \to -\infty$ as $x \to +\dots$ $f(x) \to -\infty$ as $x \to -\dots$
iv. $f(x) \dots \to +\dots$ $f(x) \dots \to -\dots$

Helping You Remember

3. What is an easy way to remember the difference between the end behavio… of even-degree and odd-degree polynomial functions?

Sample answer: Both ends of the graph of an even-degree fu… eventually keep going in the same direction. For odd-degree… the two ends eventually head in opposite directions, one upv… other downward.

Enrichment, p. 380

Approximation by Means of Polynomials

Many scientific experiments produce pairs of numbers $(x, f(x))$ that can be related by a formula. If the pairs form a function, you can fit a polynomial to the pairs in exactly one way. Consider the pairs given by the following table.

x	1	2	4	7
$f(x)$	6	11	39	−54

We will assume the polynomial is of degree three. Substitute the given values into this expression.

$f(x) = A + B(x - x_0) + C(x - x_0)(x - x_1) + D(x - x_0)(x - x_1)(x - x_2)$

You will get the system of equations shown below. You can solve this system and use the values for A, B, C, and D to find the desired polynomial.

$6 = A$
$11 = A + B(2 - 1) = A + B$
$39 = A + \dots 1) + C(4 - 1)(4 - \dots = A + 3B + 6C$

P7

Use long division or synthetic division for each of the following:

1. $(2x^2 - 5x - 3) \div (x - 3)$

2. $(x^2 - 3x - 7) \div (x + 2)$

3. $\frac{x^3-6}{x-1}$

4. $(x^3 - 6x^2 + 1) \div (x + 2)$

5. $\frac{x^5-1}{x-1}$

6. $(2x^3 - 5x^2 + 4x - 4) \div (x - 2)$

Answers:

1. $2x+1$
2. $x-5+\frac{3}{x+2}$
3. $x^2+x+1+\frac{-5}{x-1}$
4. $x^2-8x+16+\frac{-31}{x+2}$
5. $x^4+x^3+x^2+x+1$
6. $2x^2-x+2$

P9
Advanced Algebra

Name: _____
Period: _____ Date: _____

Vrite a polynomial function of least degree with integral coefficients whose zeros include the following:

1. $-5, 3i$

2. $-2, 3i$

3. $-1, 4, 3i$

4. $2, 5, -i$

5. $4i, 4, -3$

6. $9, 2i$

7. $-5, 2, -i$

Practice and Apply

Homework Help

For Exercises	See Examples
13–20	1
21–36	2
37–44	3

Extra Practice See page 843.

Use synthetic substitution to find $g(3)$ and $g(-4)$ for each function. **18. 267, 600**

13. $g(x) = x^2 - 8x + 6$ **−9, 54**
14. $g(x) = x^3 + 2x^2 - 3x + 1$ **37, −19**
15. $g(x) = x^3 - 5x + 2$ **14, −42**
16. $g(x) = x^4 - 6x - 8$ **55, 272**
17. $g(x) = 2x^3 - 8x^2 - 2x + 5$ **−19, −243**
18. $g(x) = 3x^4 + x^3 - 2x^2 + x + 12$
19. $g(x) = x^5 + 8x^3 + 2x - 15$ **450, −1559**
20. $g(x) = x^6 - 4x^4 + 3x^2 - 10$ **422, 311**

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials. **24. $x - 3$, $x - 1$**

21. $x^3 + 2x^2 - x - 2$; $x - 1$ **$x + 1$, $x + 2$**
22. $x^3 - x^2 - 10x - 8$; $x + 1$ **$x - 4$, $x + 2$**
23. $x^3 + x^2 - 16x - 16$; $x + 4$ **$x - 4$, $x + 1$**
24. $x^3 - 6x^2 + 11x - 6$; $x - 2$

p9. 368

Answers

25. $x + 3$, $x - \frac{1}{2}$ or $2x - 1$

26. $x - 1$, $x + \frac{4}{3}$ or $3x + 4$

27. $x + 7$, $x - 4$

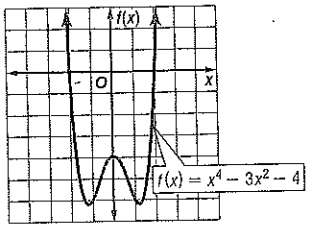
28. $x - 1$, $x + 6$

29. $x - 1$, $x^2 + 2x + 3$

30. $2x - 3$, $2x + 3$, $4x^2 + 9$

25. $2x^3 - 5x^2 - 28x + 15$; $x - 5$
26. $3x^3 + 10x^2 - x - 12$; $x + 3$
27. $2x^3 + 7x^2 - 53x - 28$; $2x + 1$
28. $2x^3 + 17x^2 + 23x - 42$; $2x + 7$
29. $x^4 + 2x^3 + 2x^2 - 2x - 3$; $x + 1$
30. $16x^5 - 32x^4 - 81x + 162$; $x - 2$

31. Use the graph of the polynomial function at the right to determine at least one binomial factor of the polynomial. Then find all the factors of the polynomial. **$x - 2$, $x + 2$, $x^2 + 1$**



32. Use synthetic substitution to show that $x - 8$ is a factor of $x^3 - 4x^2 - 29x - 24$. Then find any remaining factors.

Practice and Apply

Homework Help

For Exercises	See Examples
12–17	1
18–29, 34–41	2
30–33	3

Extra Practice See page 845.

List all of the possible rational zeros of each function.

12. $f(x) = x^3 + 6x + 2$ **±1, ±2**
13. $h(x) = x^3 + 8x + 6$ **±1, ±2, ±3, ±6**
14. $f(x) = 3x^4 + 15$
15. $n(x) = x^5 + 6x^3 - 12x + 18$
16. $p(x) = 3x^3 - 5x^2 - 11x + 3$
17. $h(x) = 9x^6 - 5x^3 + 27$

Find all of the rational zeros of each function. **18. −6, −5, 10**

18. $f(x) = x^3 + x^2 - 80x - 300$
19. $p(x) = x^3 - 3x - 2$ **−1, −1, 2**
20. $h(x) = x^4 + x^2 - 2$ **1, −1**
21. $g(x) = x^4 - 3x^3 - 53x^2 - 9x$ **0, 9**
22. $f(x) = 2x^5 - x^4 - 2x + 1$ **$\frac{1}{2}$, −1, 1**
23. $f(x) = x^5 - 6x^3 + 8x$ **0, 2, −2**
24. $g(x) = x^4 - 3x^3 + x^2 - 3x$ **0, 3**
25. $p(x) = x^4 + 10x^3 + 33x^2 + 38x + 8$
26. $p(x) = x^3 + 3x^2 - 25x + 21$ **−7, 1, 3**
27. $h(x) = 6x^3 + 11x^2 - 3x - 2$
28. $h(x) = 10x^3 - 17x^2 - 7x + 2$
29. $g(x) = 48x^4 - 52x^3 + 13x - 3$

25. −2, −4 27. $\frac{1}{2}$, $-\frac{1}{3}$, −2 28. $-\frac{1}{2}$, $\frac{1}{5}$, 2 29. $-\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{3}{4}$

Find all of the zeros of each function. **30–33. See margin.**

30. $p(x) = 6x^4 + 22x^3 + 11x^2 - 38x - 40$
31. $g(x) = 5x^4 - 29x^3 + 55x^2 - 28x$
32. $h(x) = 9x^5 - 94x^3 + 27x^2 + 40x - 12$
33. $p(x) = x^5 - 2x^4 - 12x^3 - 12x^2 - 13x - 10$

14. $\pm 1, \pm 3, \pm 5, \pm 15, \pm\frac{1}{3}, \pm\frac{5}{3}$
15. $\pm 1, \pm 2, \pm 3, \pm 6, \pm 9, \pm 18$
16. $\pm 1, \pm\frac{1}{3}, \pm 3$
17. $\pm 1, \pm\frac{1}{3}, \pm\frac{1}{9}, \pm 3, \pm 9, \pm 27$

p9. 381