PA 8-1 # 7-10 all, 13, 17, 26, 31, 47, 49

$$\begin{bmatrix} -2 & 0 & 3 & 4 \\ 3 & 1 & 5 & -1 \\ 1 & 4 & -1 & 3 \end{bmatrix}$$

(7) $a_{13} = 3$ 1st row 3rd column

(8) $a_{24} = -1$

(9) $a_{32} = 4$

(10) $a_{33} = -1$

2x3 ← same → 2x3

(13) (a) $A + B = \begin{bmatrix} -3 & 1 \\ 0 & -1 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ -2 & 1 \\ -3 & -1 \end{bmatrix} = \begin{bmatrix} -3+4 & 1+0 \\ 0+(-2) & -1+1 \\ 2+(-3) & 1+(-1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & 0 \\ -1 & 0 \end{bmatrix}$

(b) $A - B = \begin{bmatrix} -3 & 1 \\ 0 & -1 \\ 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ -2 & 1 \\ -3 & -1 \end{bmatrix} = \begin{bmatrix} -7 & 1 \\ 2 & -2 \\ 5 & 2 \end{bmatrix}$

(c) $3A = 3\begin{bmatrix} -3 & 1 \\ 0 & -1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} -9 & 3 \\ 0 & -3 \\ 6 & 3 \end{bmatrix}$

(d) $2A - 3B = 2\begin{bmatrix} -3 & 1 \\ 0 & -1 \\ 2 & 1 \end{bmatrix} - 3\begin{bmatrix} 4 & 0 \\ -2 & 1 \\ -3 & -1 \end{bmatrix} = \begin{bmatrix} -6 & 2 \\ 0 & -2 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} -12 & 0 \\ 6 & -3 \\ 9 & 3 \end{bmatrix} = \begin{bmatrix} -18 & 2 \\ 6 & -5 \\ 13 & 5 \end{bmatrix}$

(17) (a) $AB = \begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix} \cdot \begin{bmatrix} 1 & -3 \\ -2 & -4 \end{bmatrix} = \begin{bmatrix} 2(1) + 3(-2) & 2(-3) + 3(-4) \\ -1(1) + 5(-2) & -1(-3) + 5(-4) \end{bmatrix} = \begin{bmatrix} -4 & -18 \\ -11 & -17 \end{bmatrix}$

2x2 2x2

2=2 ✓

Result: 2x2

(b) $BA = \begin{bmatrix} 1 & -3 \\ -2 & -4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} 1(2) + (-3)(-1) & 1(3) + (-3)(5) \\ -2(2) + (-4)(-1) & -2(3) + (-4)(5) \end{bmatrix} = \begin{bmatrix} 5 & -12 \\ 0 & -26 \end{bmatrix}$

* Support your answer in (a)+(b) w/ N-spire.

(26) (a) $AB = \begin{bmatrix} -1 & 3 \\ 0 & 1 \\ 1 & 0 \\ -3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 5 & -6 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} (-1)(5) + 3(2) & (-1)(-6) + 3(3) \\ 0(5) + 1(2) & 0(-6) + 1(3) \\ 1(5) + 0(2) & 1(-6) + 0(3) \\ (-3)(5) + (-1)(2) & (-3)(-6) + (-1)(3) \end{bmatrix} = \begin{bmatrix} 1 & 15 \\ 2 & 3 \\ 5 & -6 \\ -17 & 15 \end{bmatrix}$

Matrix A: Matrix B:

4 x 2 2 x 2

2=2 ✓

Resulting matrix: 4x2

* Support answer with N-spire

b) BA = ?

Matrix B: Matrix A:

2x2 4x2

$2 \neq 4$

∴, Matrix multiplication is NOT defined for BA.

(31)

$$\begin{bmatrix} 2 & a-1 \\ 2 & 3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ b+2 & 3 \\ -1 & 2 \end{bmatrix}$$

Set corresponding element = to each other.

$a - 1 = -3$ $\quad\quad$ $2 = b + 2$

$\boxed{a = -2}$ $\quad\quad$ $\boxed{0 = b}$

(47)

$$[a_{ij}] = A = \underbrace{\begin{bmatrix} 100 & 60 \\ 120 & 70 \\ 200 & 120 \end{bmatrix}}_{3\times 2}$$

rows: # of dozens of type (1 large, 2 x-large, 3 jumbo) eggs

columns: the grocery store sold to (not provided)

$$B = \underbrace{\begin{bmatrix} \$0.80 \\ \$0.85 \\ \$1.00 \end{bmatrix}}_{3\times 1} = [b_{i1}]$$

(a) $B^T A = [0.80 \quad 0.85 \quad 1.00] \cdot \begin{bmatrix} 100 & 60 \\ 120 & 70 \\ 200 & 120 \end{bmatrix}$

"B transpose"

$1\times \underbrace{3 \quad 3}\times 2$

$3 = 3$ ✓

Result: 1×2

$= [0.80(100) + 0.85(120) + 1.00(200) \quad 0.80(60) + 0.85(70) + 1.00(120)]$

$= [\$382 \quad \$227.50]$, 1×2 matrix

$\therefore$, $B^T A = [\$382 \quad \$227.50]$

$\downarrow \quad\quad \downarrow$

$b_{11} \quad\quad b_{12}$

(b) $b_{11} = \$382$ represents the income Happy Farms made at one of the grocery stores selling all three types of eggs

Similarly, b_{12} represents the income Happy Farms made at the other grocery store selling all three types of eggs.

(49) a) Total revenue $= AB^T = BA^T$

$1\times 4 \; 4\times 1$

$4 = 4$ ✓

1×1 :)

$$AB^T = \underbrace{[\$398 \quad \$598 \quad \$798 \quad \$998]}_{1\times 4} \cdot \underbrace{\begin{bmatrix} 35 \\ 25 \\ 20 \\ 10 \end{bmatrix}}_{4\times 1} = [\$54,820] = \overset{B}{[35 \quad 25 \quad 20 \quad 10]} \cdot \overset{A^T}{\begin{bmatrix} \$398 \\ \$598 \\ \$798 \\ \$998 \end{bmatrix}}$$

b) Profit = Total Revenue − Total cost

(money earned from sales) $\quad$ (involved in making furniture)

$$= AB^T - CB^T \text{ (or } BC^T) = AB^T - [\$199 \quad \$268 \quad \$500 \quad \$670] \cdot \begin{bmatrix} 35 \\ 25 \\ 20 \\ 10 \end{bmatrix} = [\$54,820] - [\$30,365] = [\$24,455]$$

PA 8-2 p. 539 # 33, 35, 37, 41, 44, 45, 62, 66

(33) Need to show: $AB = I_n$ and $BA = I_n$ → Recall: Matrix multiplication is ~~not~~ not commutative

$$AB = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 0.8 & -0.2 \\ -0.6 & 0.4 \end{bmatrix} = \begin{bmatrix} 2(0.8)+1(-0.6) & 2(-0.2)+1(0.4) \\ 3(0.8)+4(-0.6) & 3(-0.2)+4(0.4) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2 \checkmark$$

$$BA = \begin{bmatrix} 0.8 & -0.2 \\ -0.6 & 0.4 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 0.8(2)+(-0.2)(3) & 0.8(1)+(-0.2)(4) \\ -0.6(2)+0.4(3) & -0.6(1)+0.4(4) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2 \checkmark$$

$\therefore$, since $AB = BA = I_2$, the matrices are inverses of eachother.

(35) Let $A = \begin{bmatrix} 2 & 3 \\ 2 & 2 \end{bmatrix}$ $\quad$ $A^{-1} = \frac{1}{|A|} \cdot$ Adjugate Matrix

$$A^{-1} = \frac{1}{2(2)-3(2)} \cdot \begin{bmatrix} 2 & -3 \\ -2 & 2 \end{bmatrix} = \frac{1}{-2} \cdot \begin{bmatrix} 2 & -3 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} \frac{2}{-2} & \frac{3}{2} \\ \frac{-2}{-2} & \frac{2}{-2} \end{bmatrix} = \boxed{\begin{bmatrix} -1 & \frac{3}{2} \\ 1 & -1 \end{bmatrix} \text{ or } \begin{bmatrix} -1 & 1.5 \\ 1 & -1 \end{bmatrix}}$$

(37) Let $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ $\quad$ $\boxed{A^{-1} \text{ D.N.E or no inverse}}$ $\quad$ A is a singular matrix

(41)

$$\left| \begin{matrix} 2 & 1 & 1 \\ -1 & 0 & 2 \\ 1 & 3 & -1 \end{matrix} \right| \begin{matrix} 2 & 1 \\ -1 & 0 \\ 1 & 3 \end{matrix} = 0 + 2 + (-3) - 0 - 12 - 1 = \boxed{-14}$$

Multiply #s on the diagonal

2nd

Then subtract

Multiply #'s on the diagonal

1st

Then add

(44) Solve for X. (Matrix X) $\quad$ Let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad$ X must be 2×2 matrix since A, B are 2×2 matrices

$2X + A = B$

$$2\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 2a-1 & 2b+2 \\ 2c & 2d+3 \end{bmatrix} = \begin{bmatrix} 1 & 4 \\ 1 & -1 \end{bmatrix}$$

$2a - 1 = 1$ $\quad$ $2b + 2 = 4$

$2a = 2$ $\quad$ $2b = 2$

$\boxed{a = 1}$ $\quad$ $\boxed{b = 1}$

$2c = 1$ $\quad$ $2d + 3 = -1$

$\boxed{c = \frac{1}{2}}$ $\quad$ $2d = -4$

$\boxed{d = -2}$

$$\boxed{\therefore, X = \begin{bmatrix} 1 & 1 \\ \frac{1}{2} & -2 \end{bmatrix}}$$

OR

$2X + A = B$

$2X = B - A$

$X = \frac{B-A}{2} = \frac{1}{2}(B-A)$

$$X = \frac{1}{2}\left(\begin{bmatrix} 1 & 4 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} -1 & 2 \\ 0 & 3 \end{bmatrix} \right)$$

$$= \frac{1}{2} \cdot \begin{bmatrix} 2 & 2 \\ 1 & -4 \end{bmatrix}$$

$$\boxed{= \begin{bmatrix} 1 & 1 \\ \frac{1}{2} & -2 \end{bmatrix}} \checkmark$$

(45) Matrix gives road mileage between cities

$$\begin{array}{c} \\ A \\ B \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 689 & 774 & 1406 \\ 689 & 0 & 371 & 1685 \\ 774 & 371 & 0 & 1340 \\ 1406 & 1685 & 1340 & 0 \end{bmatrix} \end{array}$$

a) Take, for example, the entry in the 4th row and the 2nd column: $a_{42} = 1685$

The reason why the entry in 2nd row and 4th column is the <u>same</u> is because each entry represents the distance between the same two cities, namely the distance between Baltimore (B) and Denver (D).

b) The distance between a city and itself is 0.

(62) $\boxed{\text{FALSE.}}$ A square matrix has an inverse $\Longleftrightarrow$ detrminant is NOT zero

So if a square matrix B has a determinant that's 0, then matrix B has no inverse.

(66) Let $B = \begin{bmatrix} 2 & 7 \\ 1 & 4 \end{bmatrix}$

$$B^{-1} = \frac{1}{2(4) - 7(1)} \cdot \begin{bmatrix} 4 & -7 \\ -1 & 2 \end{bmatrix}$$

$$= \frac{1}{1} \cdot \begin{bmatrix} 4 & -7 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -7 \\ -1 & 2 \end{bmatrix}$$

$\boxed{\therefore \text{ (E)}}$

Check:

$$BB^{-1} = \begin{bmatrix} 2 & 7 \\ 1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 & -7 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2(4)+7(-1) & 2(-7)+7(2) \\ 1(4)+4(-1) & 1(-7)+4(2) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \checkmark$$

$$B^{-1}B = \begin{bmatrix} 4 & -7 \\ -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 7 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 4(2)+(-7)(1) & 4(7)+(-7)(4) \\ -1(2)+2(1) & -1(7)+2(4) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \checkmark$$

We must verify both ways b/c matrix multiplication is not commutative

PA 8-3 p.553 #25, 49, 51, 53, 55, 83, 85 (using Inverse Matrix Methods)

(25)
$$\begin{cases} 2x - 3y + z = 1 \\ -x + y - 4z = -3 \\ 3x - z = 2 \end{cases}$$

Missing y

Set up: $AX=B$

$$\begin{bmatrix} 2 & -3 & 1 \\ -1 & 1 & -4 \\ 3 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}$$

Fill in with 0

Solve: $X=A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & -3 & 1 \\ -1 & 1 & -4 \\ 3 & 0 & -1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}}_{\text{calculator}}$$

3x3 3x1, 3=3✓, 3x1

$$= \begin{bmatrix} 15/17 \\ 8/17 \\ 11/17 \end{bmatrix}$$

$\boxed{\therefore,\ x = \frac{15}{17},\ y = \frac{8}{17},\ z = \frac{11}{17}}$

or

$\boxed{\left(\frac{15}{17}, \frac{8}{17}, \frac{11}{17}\right)}$

(49)
$$\begin{cases} 2x - 3y = -13 \\ 4x + y = -5 \end{cases}$$

Set up: $AX=B$

$$\begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -13 \\ -5 \end{bmatrix}$$

Solve: $X=A^{-1}B$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -13 \\ -5 \end{bmatrix}$$

$$= \frac{1}{2(1)-(-3)(4)} \cdot \begin{bmatrix} 1 & 3 \\ -4 & 2 \end{bmatrix} \cdot \begin{bmatrix} -13 \\ -5 \end{bmatrix}$$

$$= \frac{1}{14} \cdot \begin{bmatrix} 1 & 3 \\ -4 & 2 \end{bmatrix} \cdot \begin{bmatrix} -13 \\ -5 \end{bmatrix}$$

2x2 2x1, 2=2✓, Result: 2x1

$$= \frac{1}{14} \cdot \begin{bmatrix} 1(-13) + 3(-5) \\ (-4)(-13) + 2(-5) \end{bmatrix}$$

$$= \frac{1}{14} \cdot \begin{bmatrix} -28 \\ 42 \end{bmatrix}$$

$\frac{1}{14} \cdot -28$, $\frac{1}{14} \cdot 42$

$$= \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$\boxed{\therefore,\ x = -2,\ y = 3}$

or

$\boxed{(-2, 3)}$

(51)
$$\begin{cases} 2x - y + z = -6 \\ x + 2y - 3z = 9 \\ 3x - 2y + z = -3 \end{cases}$$

Set up: $AX=B$

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -6 \\ 9 \\ -3 \end{bmatrix}$$

Solve: $X=A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & -2 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -6 \\ 9 \\ -3 \end{bmatrix}}_{\text{calculator}}$$

$$= \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}$$

$\boxed{\therefore,\ x = -2,\ y = -5,\ z = -7}$

or

$\boxed{(-2, -5, -7)}$

(53)
$$\begin{cases} 2x - y + z + w = -3 \\ x + 2y - 3z + w = 12 \\ 3x - y - z + 2w = 3 \\ -2x + 3y + z - 3w = -3 \end{cases}$$

$\boxed{(-1, 2, -2, 3)}$

or

$\boxed{\therefore\ x = -1,\ y = 2,\ z = -2,\ w = 3}$

set up: $AX = B$

$$\begin{bmatrix} 2 & -1 & 1 & 1 \\ 1 & 2 & -3 & 1 \\ 3 & -1 & -1 & 2 \\ -2 & 3 & 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} -3 \\ 12 \\ 3 \\ -3 \end{bmatrix}$$

Solve: $X=A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 2 & -1 & 1 & 1 \\ 1 & 2 & -3 & 1 \\ 3 & -1 & -1 & 2 \\ -2 & 3 & 1 & -3 \end{bmatrix}^{-1} \cdot \begin{bmatrix} -3 \\ 12 \\ 3 \\ -3 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ -2 \\ 3 \end{bmatrix}$$

(55)
$$\begin{cases} 2x - y = 10 \\ x - z = -1 \\ y + z = -9 \end{cases}$$

Fill in missing variables w/ zeros ☺

Set up: $AX=B$

$$\begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -1 \\ -9 \end{bmatrix}$$

$\boxed{\therefore,\ x = 0,\ y = -10,\ z = 1}$

or

$\boxed{(0, -10, 1)}$

Solve: $X=A^{-1}B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 10 \\ -1 \\ -9 \end{bmatrix} = \begin{bmatrix} 0 \\ -10 \\ 1 \end{bmatrix}$$

(83) $f(x) = ax^2 + bx + c$; $\underset{x \quad f(x)}{(-1, 3)}$, $\underset{x \quad f(x)}{(1, -3)}$, $\underset{x \quad f(x)}{(2, 0)}$ (Input, Output)

$$\left.\begin{array}{l} f(-1) = a(-1)^2 + b(-1) + c = a - b + c = 3 \\ f(1) = a(1)^2 + b(1) + c = a + b + c = -3 \\ f(2) = a(2)^2 + b(2) + c = 4a + 2b + c = 0 \end{array}\right\} \Rightarrow \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix} \Rightarrow$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 3 \\ -3 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ -2 \end{bmatrix}$$

$\therefore$, $a = 2$, $b = -3$, $c = -2$ or $(2, -3, -2)$

$$\boxed{\therefore,\ f(x) = 2x^2 - 3x - 2}$$

(85) $f(x) = ax^2 + bx + c$; $(-1, -4)$, $(1, -2)$

$$\left.\begin{array}{l} f(-1) = a(-1)^2 + b(-1) + c = a - b + c = -4 \\ f(1) = a(1)^2 + b(1) + c = a + b + c = -2 \end{array}\right\} \Rightarrow \begin{array}{l} ① \ a - \cancel{b} + c = -4 \\ ② \ a + \cancel{b} + c = -2 \\ \hline 2a + 2c = -6 \end{array}$$

(Factor 2)

$$\frac{2(a+c)}{2} = \frac{-6}{2}$$

$$\underbrace{a + c = -3}$$

we can solve this equation for a or for c.

* Solve for a.

$a + c = -3$

$a = \boxed{-3 - c}$ sub in for ① or ②

② $a + b + c = -2$

$(-3 - c) + b + c = -2$

$-3 - \cancel{c} + b + \cancel{c} = -2$

$$\begin{array}{r} -3 + b = -2 \\ +3 \qquad +3 \\ \hline b = 1 \end{array}$$

$\therefore$, $a = -3 - c$

$b = 1$ or $(-3-c, 1, c)$

$c = c$

$$\boxed{\therefore,\ f(x) = (-3-c)x^2 + x + c, \text{ for any } c.}$$

OR

* Solve for c.

$a + c = -3$

$c = \boxed{-3 - a}$ sub in for ① or ②

② $a + b + c = -2$

$a + b + (-3 - a) = -2$

$\cancel{a} + b - 3 - \cancel{a} = -2$

$$\begin{array}{r} b - 3 = -2 \\ +3 \quad +3 \\ \hline b = 1 \end{array}$$

$\therefore$, $a = a$

$b = 1$

$c = -3 - a$

or

$(a, 1, -3-a)$

$$\boxed{\therefore,\ f(x) = ax^2 + x + (-3-a), \text{ for any } a.}$$

PA8-4 p.553 #27, 43, 44, 59, 61

(27)
$$\begin{cases} 2x-5y+z-w=-3 \\ x-2z+w=4 \\ 2y-3z-w=5 \end{cases}$$

Fill in missing variables w/zero's in matrix

$$\left[\begin{array}{cccc|c} 2 & -5 & 1 & -1 & -3 \\ 1 & 0 & -2 & 1 & 4 \\ 0 & 2 & -3 & -1 & 5 \end{array}\right]$$

(43)
$$\begin{cases} x+y-3z=1 \\ x-z-w=2 \\ 2x+y-4z-w=3 \end{cases}$$

$$\text{rref}\left(\left[\begin{array}{cccc|c} 1 & 1 & -3 & 0 & 1 \\ 1 & 0 & -1 & -1 & 2 \\ 2 & 1 & -4 & -1 & 3 \end{array}\right]\right) = \begin{array}{c} \begin{array}{cccc} x & y & z & w \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & -1 & -1 & 2 \\ 0 & 1 & -2 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array} \begin{array}{l} \to x-z-w=2 \\ \to y-2z+w=-1 \\ \to 0z=0 \to 0=0 \\ \quad 0w=0 \quad \text{TRUE} \end{array}$$

Any # will satisfy this equation, so z, w can be any #s

Inf. many solutions

$$\begin{array}{rcl} x-z-w &=& 2 \\ +z+w & & +z+w \\ \hline x &=& 2+z+w \end{array} \qquad \begin{array}{rcl} y-2z+w &=& -1 \\ +2z-w & & +2z-w \\ \hline y &=& -1+2z-w \end{array}$$

$$\boxed{\therefore,\ (\overset{x}{2+z+w},\ \overset{y}{-1+2z-w},\ \overset{z}{z},\ \overset{w}{w}) \text{ for any } z, w.}$$

(44)
$$\begin{cases} x-y-z+2w=-3 \\ 2x-y-2z+3w=-3 \\ x-2y-z+3w=-6 \end{cases}$$

$$\text{rref}\left(\left[\begin{array}{cccc|c} 1 & -1 & -1 & 2 & -3 \\ 2 & -1 & -2 & 3 & -3 \\ 1 & -2 & -1 & 3 & -6 \end{array}\right]\right) = \begin{array}{c} \begin{array}{cccc} x & y & z & w \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array} \begin{array}{l} \to x-z+w=0 \\ \to y-w=3 \\ \to 0=0\text{, TRUE} \end{array}$$

↳ z, w can be any #

$x-z+w=0 \Rightarrow x=z-w$

$y-w=3 \Rightarrow y=3+w$

$$\boxed{\therefore,\ (\overset{x}{z-w},\ \overset{y}{3+w},\ \overset{z}{z},\ \overset{w}{w}) \text{ for any } z, w.}$$

(59)
$$\begin{cases} x-y+z=6 \\ x+y+2z=-2 \end{cases}$$

$$\text{rref}\left(\left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 1 & 1 & 2 & -2 \end{array}\right]\right) = \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & \frac{3}{2} & 2 \\ 0 & 1 & \frac{1}{2} & -4 \end{array}\right] \end{array} \begin{array}{l} \to x+\frac{3}{2}z=2 \\ \to y+\frac{1}{2}z=-4 \end{array}$$

$x+\frac{3}{2}z=2 \Rightarrow x=2-\frac{3}{2}z$, where z can be any #

$y+\frac{1}{2}z=-4 \Rightarrow y=-4-\frac{1}{2}z$

$$\boxed{\therefore,\ (\overset{x}{2-\tfrac{3}{2}z},\ \overset{y}{-4-\tfrac{1}{2}z},\ \overset{z}{z}) \text{ for any } z.}$$

(61)
$$\begin{cases} 2x+y+z+4w=-1 \\ x+2y+z+w=1 \\ x+y+z+2w=0 \end{cases}$$

$$\text{rref}\left(\left[\begin{array}{cccc|c} 2 & 1 & 1 & 4 & -1 \\ 1 & 2 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 & 0 \end{array}\right]\right) = \begin{array}{c} \begin{array}{cccc} x & y & z & w \end{array} \\ \left[\begin{array}{cccc|c} 1 & 0 & 0 & 2 & -1 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right] \end{array} \begin{array}{l} \to x+2w=-1 \\ \to y-w=1 \\ \to z+w=0 \end{array}$$

$x+2w=-1 \Rightarrow x=-1-2w$

$y-w=1 \Rightarrow y=1+w$, where w can be any #

$z+w=0 \Rightarrow z=-w$

$$\boxed{\therefore,\ (\overset{x}{-1-2w},\ \overset{y}{1+w},\ \overset{z}{-w},\ \overset{w}{w}), \text{ for any } w.}$$

PA 8-5 p. 554 #67, 68, 69-77 odd

(67) $$\frac{x+22}{(x+4)(x-2)} = \frac{A}{x+4} + \frac{B}{x-2}$$ LCD: $(x+4)(x-2)$

$$\cancel{(x+4)(x-2)}\cdot\frac{x+22}{\cancel{(x+4)(x-2)}} = \frac{A}{\cancel{x+4}}\cdot\cancel{(x+4)}(x-2) + \frac{B}{\cancel{x-2}}\cdot(x+4)\cancel{(x-2)}$$

$$x+22 = A(x-2) + B(x+4)$$

$$x+22 = Ax - 2A + Bx + 4B$$

$$x+22 = Ax + Bx - 2A + 4B$$

$$1x + 22 = (A+B)x - 2A + 4B$$

$$\begin{cases} 1 = A+B \\ 22 = -2A+4B \end{cases} \Rightarrow \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}\cdot\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 1 \\ 22 \end{bmatrix}$$

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}^{-1}\cdot\begin{bmatrix} 1 \\ 22 \end{bmatrix}$$

Inverse matrix method

$$= \frac{1}{1(4)-1(-2)}\cdot\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}\cdot\begin{bmatrix} 1 \\ 22 \end{bmatrix}$$

$$= \frac{1}{6}\cdot\begin{bmatrix} 4(1)+(-1)(22) \\ 2(1)+(1)(22) \end{bmatrix}$$

$$= \frac{1}{6}\cdot\begin{bmatrix} -18 \\ 24 \end{bmatrix}$$

$$= \begin{bmatrix} -3 \\ 4 \end{bmatrix}$$

$\therefore$, $A=-3$, $B=4$

$$\boxed{\therefore,\ \frac{x+22}{(x+4)(x-2)} = \frac{-3}{x+4} + \frac{4}{x-2}}$$

(68) $$\frac{x-3}{x(x+3)} = \frac{A}{x+3} + \frac{B}{x}$$ LCD: $x(x+3$

$$\cancel{x(x+3)}\cdot\frac{x-3}{\cancel{x(x+3)}} = \frac{A}{\cancel{x+3}}\cdot x\cancel{(x+3)} + \frac{B}{\cancel{x}}\cdot\cancel{x}(x+3)$$

$$x-3 = Ax + B(x+3)$$

$$x-3 = Ax + Bx + 3B$$

$$1x - 3 = (A+B)x + 3B$$

$$\begin{cases} 1 = A+B \\ -3 = 3B \end{cases} \Rightarrow \begin{matrix} 1 = A+B \\ -3 = 0A+3B \end{matrix} \Rightarrow$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}\cdot\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \end{bmatrix} \Rightarrow \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 3 \end{bmatrix}^{-1}\cdot\begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} A \\ B \end{bmatrix} = \frac{1}{1(3)-1(0)}\cdot\begin{bmatrix} 3 & -1 \\ 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

$$= \frac{1}{3}\cdot\begin{bmatrix} 3(1)+(-1)(-3) \\ 0(1)+1(-3) \end{bmatrix}$$

$$= \frac{1}{3}\cdot\begin{bmatrix} 6 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$ $\therefore$, $A=2$, $B=-1$

$$\boxed{\therefore,\ \frac{x-3}{x(x+3)} = \frac{2}{x+3} + \frac{-1}{x}}$$

(69) $$\frac{2}{(x-5)(x-3)} = \frac{A}{x-5} + \frac{B}{x-3}$$ LCD: $(x-5)(x-3)$

$$\cancel{(x-5)(x-3)}\cdot\frac{2}{\cancel{(x-5)(x-3)}} = \frac{A}{\cancel{x-5}}\cdot\cancel{(x-5)}(x-3) + \frac{B}{\cancel{x-3}}\cdot(x-5)\cancel{(x-3)}$$

$$2 = A(x-3) + B(x-5)$$

Let $x=3$.

$$2 = A\cancel{(3-3)} + B(3-5)$$

$$\frac{2}{-2} = \frac{-2B}{-2}$$

$$-1 = B$$

Let $x=5$.

$$2 = A(5-3) + \cancel{B(5-5)}$$

$$\frac{2}{2} = \frac{2A}{2} +$$

$$1 = A$$

$$\boxed{\therefore,\ \frac{2}{(x-5)(x-3)} = \frac{1}{x-5} + \frac{-1}{x-3}}$$

Algebraic confirmation:

$$\frac{1}{x-5} + \frac{-1}{x-3}$$

$$= \frac{1}{(x-5)}\cdot\frac{(x-3)}{(x-3)} + \frac{-1}{(x-3)}\cdot\frac{(x-5)}{(x-5)}$$

$$= \frac{x-3-x+5}{(x-5)(x-3)}$$

$$= \frac{2}{(x-5)(x-3)} \checkmark$$

(71) $$\frac{4}{x^2-1} = \frac{4}{(x+1)(x-1)} = \frac{A}{x+1} + \frac{B}{x-1}$$

LCD: $(x+1)(x-1)$

$$\cancel{(x+1)(x-1)}\cdot\frac{4}{\cancel{(x+1)(x-1)}} = \cancel{(x+1)}(x-1)\cdot\frac{A}{\cancel{x+1}} + (x+1)\cancel{(x-1)}\cdot\frac{B}{\cancel{x-1}}$$

$$4 = A(x-1) + B(x+1)$$

$$4 = \underline{A}x - A + \underline{B}x + B$$

$$4 = Ax + Bx - A + B$$

Notice no linear term

$$4 = (A+B)x - A + B$$

$$0x + 4 = (A+B)x - A + B$$

$$\begin{cases} 0 = A + B \\ 4 = -A + B \end{cases} \Rightarrow 4 = 2B$$

$\therefore$, $B = 2$ and $A = -2$

$$\boxed{\therefore,\ \frac{4}{x^2-1} = \frac{-2}{x+1} + \frac{2}{x-1}}$$

(73) $$\frac{2}{x^2+2x} = \frac{2}{x(x+2)} = \frac{A}{x} + \frac{B}{x+2}$$

LCD: $x(x+2)$

$$\frac{2}{\cancel{x(x+2)}}\cdot\cancel{x(x+2)} = \frac{A}{\cancel{x}}\cdot\cancel{x}(x+2) + \frac{B}{\cancel{x+2}}\cdot x\cancel{(x+2)}$$

$$2 = A(x+2) + Bx$$

Let $x = -2$.

$$2 = A\cancel{(-2+2)} + B(-2)$$

$$2 = -2B$$

$$-1 = B$$

Let $x = 0$.

$$2 = A(0+2) + \cancel{B(0)}$$

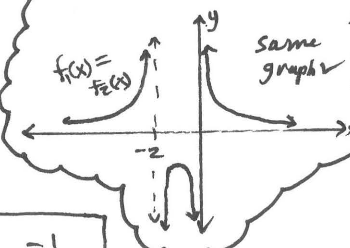
$$2 = 2A$$

$$1 = A$$

Graphical support:

Graph $f_1(x) = \frac{1}{x} + \frac{-1}{x+2}$

Graph $f_2(x) = \frac{2}{x^2+2x}$



$$\boxed{\therefore,\ \frac{2}{x^2+2x} = \frac{1}{x} + \frac{-1}{x+2}}$$

(75) $$\frac{-x+10}{x^2+x-12} = \frac{-x+10}{(x+4)(x-3)} = \frac{A}{x+4} + \frac{B}{x-3}$$

LCD: $(x+4)(x-3)$

$$\frac{-x+10}{\cancel{(x+4)(x-3)}}\cdot\cancel{(x+4)(x-3)} = \frac{A}{\cancel{x+4}}\cancel{(x+4)}(x-3) + \frac{B}{\cancel{x-3}}(x+4)\cancel{(x-3)}$$

$$-x + 10 = A(x-3) + B(x+4)$$

$$-x + 10 = Ax - 3A + Bx + 4B$$

$$-x + 10 = Ax + Bx - 3A + 4B$$

$$\underline{-1}x + 10 = (\underline{A+B})x - 3A + 4B$$

Multiply eq. by 3

$$\begin{cases} -1 = A + B \\ 10 = -3A + 4B \end{cases} \Rightarrow \begin{array}{r} -3 = \cancel{3A} + 3B \\ 10 = \cancel{-3A} + 4B \\ \hline 7 = 7B \end{array}$$

$$1 = B$$

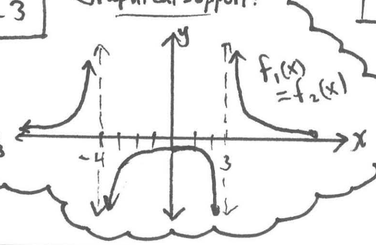
$$-1 = A + B$$
$$-1 = A + 1$$
$$\underline{-1 \qquad -1}$$
$$-2 = A$$

$$\boxed{\therefore,\ \frac{-x+10}{x^2+x-12} = \frac{-2}{x+4} + \frac{1}{x-3}}$$

Graphical support:

Graph $f_1(x) = \frac{-x+10}{x^2+x-12}$

Graph $f_2(x) = \frac{-2}{x+4} + \frac{1}{x-3}$



(77) LCD: $(2x-1)(x+3)$

$$\frac{x+17}{\underbrace{2x^2+5x-3}_{\text{Multiply}}} = \frac{x+17}{(2x-1)(x+3)} = \frac{A}{2x-1} + \frac{B}{x+3}$$

$-6x^2$

$6x \cdot -1x$

$6x - 1x = 5x$ ✓

$2x^2 + 5x - 3$

$\underline{2x^2 + 6x}\ \underline{-1x - 3}$

$2x(x+3) - 1(x+3)$

$(x+3)(2x-1)$

Factored ✓

$$\frac{x+17}{\cancel{(2x-1)(x+3)}}\cdot\cancel{(2x-1)(x+3)} = \frac{A}{\cancel{2x-1}}\cancel{(2x-1)}(x+3) + \frac{B}{\cancel{(x+3)}}(2x-1)\cancel{(x+3)}$$

$$x + 17 = A(x+3) + B(2x-1)$$

$$x + 17 = \underline{A}x + 3A + 2\underline{B}x - B$$

$$x + 17 = Ax + 2Bx + 3A - B$$

$$\underline{1}x + 17 = (\underline{A+2B})x + 3A - B$$

Mult eq. by -3

$$\begin{cases} 1 = A + 2B \\ 17 = 3A - B \end{cases} \Rightarrow \begin{array}{r} -3 = \cancel{-3A} - 6B \\ 17 = \cancel{3A} - B \\ \hline \frac{14}{-7} = \frac{-7B}{-7} \end{array}$$

$$-2 = B$$

$$1 = A + 2(-2)$$
$$1 = A - 4$$
$$\underline{+4 \qquad +4}$$
$$5 = A$$

$$\boxed{\therefore,\ \frac{x+17}{2x^2+5x-3} = \frac{5}{2x-1} + \frac{-2}{x+3}}$$

DATE: __________

Partial Fractions

Find the partial fraction decomposition for each rational expression.

1. $\frac{6x}{x^2-9}$

$\frac{6x}{(x+3)(x-3)} = \frac{A}{x+3} + \frac{B}{x-3}$ LCD: $(x+3)(x-3)$

$6x = A(x-3) + B(x+3)$

$6x = Ax - 3A + Bx + 3B$

$6x = Ax + Bx - 3A + 3B$

$6x = (A+B)x - 3A + 3B$

$6x + 0 = (A+B)x - 3A + 3B$

$\begin{cases} 6 = A+B \\ 0 = -3A+3B \end{cases} \Rightarrow \begin{matrix} 18 = 3A+3B \\ 0 = -3A+3B \\ \hline 18 = 6B \Rightarrow \boxed{3=B}, \boxed{A=3} \end{matrix}$

$\boxed{\therefore \frac{6x}{x^2-9} = \frac{3}{x+3} + \frac{3}{x-3}}$

2. $\frac{-8}{x^2-4x}$

$\frac{-8}{x(x-4)} = \frac{A}{x} + \frac{B}{x-4}$ LCD: $x(x-4)$

$-8 = A(x-4) + Bx$

Let $x = 4$.

$-8 = \cancel{A(4-4)} + B \cdot 4$

$-8 = 4B$

$\boxed{-2 = B}$

Let $x = 0$.

$-8 = A(0-4) + \cancel{B \cdot 0}$

$-8 = -4A$

$\boxed{2 = A}$

$\boxed{\therefore \frac{-8}{x^2-4x} = \frac{2}{x} - \frac{2}{x-4}}$

3. $\frac{x-16}{x^2+x-2}$

$\frac{x-16}{(x+2)(x-1)} = \frac{A}{x+2} + \frac{B}{x-1}$ LCD: $(x+2)(x-1)$

$x-16 = A(x-1) + B(x+2)$

$x-16 = Ax - A + Bx + 2B$

$x-16 = Ax + Bx - A + 2B$

$1x - 16 = (A+B)x - A + 2B$

$\begin{cases} 1 = A+B \\ -16 = -A+2B \end{cases}$

$-15 = 3B$

$\boxed{-5 = B}$

$1 = A+B$

$1 = A - 5$

$+5 \quad +5$

$\boxed{6 = A}$

$\boxed{\therefore \frac{x-16}{x^2+x-2} = \frac{6}{x+2} - \frac{5}{x-1}}$

4. $\frac{-2x-14}{x^2+6x+8}$

LCD: $(x+4)(x+2)$

$\frac{-2x-14}{(x+4)(x+2)} = \frac{A}{x+4} + \frac{B}{x+2}$

$-2x-14 = A(x+2) + B(x+4)$

Let $x = -2$.

$-2(-2) - 14 = \cancel{A(-2+2)} + B(-2+4)$

$-10 = 2B$

$\boxed{-5 = B}$

$\boxed{\therefore \frac{-2x-14}{x^2+6x+8} = \frac{3}{x+4} + \frac{-5}{x+2}}$

Let $x = -4$

$-2(-4) - 14 = A(-4+2) + \cancel{B(-4+4)}$

$-6 = -2A$

$\boxed{3 = A}$

5. $\frac{3x^2+2x+2}{(x^2+1)^2}$

$\frac{3x^2+2x+2}{(x^2+1)^2} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2}$ LCD: $(x^2+1)^2$

$3x^2+2x+2 = (Ax+B)(x^2+1) + Cx + D$

$3x^2+2x+2 = Ax^3 + Ax + Bx^2 + B + Cx + D$

$3x^2+2x+2 = Ax^3 + Bx^2 + Ax + Cx + B + D$

$3x^2+2x+2 = Ax^3 + Bx^2 + (A+C)x + B + D$

$0x^3 + 3x^2 + 2x + 2 = Ax^3 + Bx^2 + (A+C)x + B + D$

$\begin{cases} \boxed{0 = A} \\ \boxed{3 = B} \\ 2 = A + C \Rightarrow 2 = 0 + C \Rightarrow \boxed{2 = C} \\ 2 = B + D \Rightarrow \boxed{-1 = D} \end{cases}$

$\therefore \frac{3x^2+2x+2}{(x^2+1)^2} = \frac{0x+3}{x^2+1} + \frac{2x-1}{(x^2+1)^2}$

$\boxed{\frac{3x^2+2x+2}{(x^2+1)^2} = \frac{3}{x^2+1} + \frac{2x-1}{(x^2+1)^2}}$

6. $\frac{x^2-2x+1}{(x-2)^3}$

$\frac{x^2-2x+1}{(x-2)^3} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)^3}$

LCD: $(x-2)^3$

$x^2-2x+1 = A[(x-2)^2] + B(x-2) + C$

$x^2-2x+1 = A(x^2-4x+4) + Bx - 2B + C$

$x^2-2x+1 = Ax^2 - 4Ax + 4A + Bx - 2B + C$

$x^2-2x+1 = Ax^2 - 4Ax + Bx + 4A - 2B + C$

$1x^2 - 2x + 1 = Ax^2 + (-4A+B)x + 4A - 2B + C$

$\begin{cases} \boxed{1 = A} \\ -2 = -4A + B \Rightarrow \boxed{B = 2} \\ 1 = 4A - 2B + C \Rightarrow 1 = 4(1) - 2(2) + C \end{cases}$

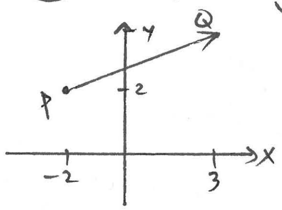
$1 = \cancel{4} - \cancel{4} + C$

$\boxed{1 = C}$

$\boxed{\therefore \frac{x^2-2x+1}{(x-2)^3} = \frac{1}{x-2} + \frac{2}{(x-2)^2} + \frac{1}{(x-2)^3}}$

PA 8-7 p. 464 # 5–19 odd

(5) $\overrightarrow{PQ}$ $\quad \underbrace{P = (\overset{x_1}{-2}, \overset{y_1}{2})}_{\text{initial pt. (tail)}}$, $\underbrace{Q = (\overset{x_2}{3}, \overset{y_2}{4})}_{\text{terminal pt. (head)}}$



Component form:

$\overrightarrow{PQ} = \langle x_2 - x_1, y_2 - y_1 \rangle$

$= \langle 3-(-2), 4-2 \rangle$

$= \boxed{\langle \underset{v_1}{5}, \underset{v_2}{2} \rangle}$

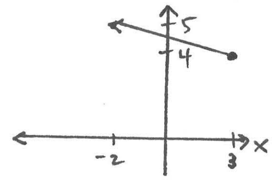
Don't forget!
Head – Tail

Magnitude: $|\overrightarrow{PQ}| = \sqrt{v_1^2 + v_2^2}$

$= \sqrt{5^2 + 2^2}$

$= \boxed{\sqrt{29}}$

(7) $\overrightarrow{QR}$ $\quad \underbrace{Q = (\overset{x_1}{3}, \overset{y_1}{4})}_{\text{initial pt.}}$, $\underbrace{R = (\overset{x_2}{-2}, \overset{y_2}{5})}_{\text{terminal pt.}}$



Component form:

$\overrightarrow{QR} = \langle -2-3, 5-4 \rangle$

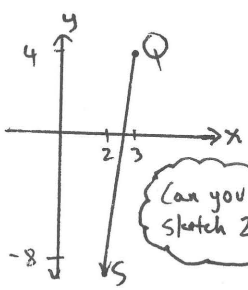
$= \boxed{\langle \underset{v_1}{-5}, \underset{v_2}{1} \rangle}$

Magnitude: $\overrightarrow{QR} = \sqrt{(-5)^2 + 1^2}$

$= \sqrt{25 + 1}$

$= \boxed{\sqrt{26}}$

(9) $2\overrightarrow{QS}$ $\quad \underbrace{Q = (\overset{x_1}{3}, \overset{y_1}{4})}_{\text{initial pt.}}$, $\underbrace{S = (\overset{x_2}{2}, \overset{y_2}{-8})}_{\text{terminal pt.}}$



Can you sketch $2\overrightarrow{QS}$?

Component form:

$\overrightarrow{QS} = \langle 2-3, -8-4 \rangle$

$= \langle -1, -12 \rangle$

$\therefore 2\overrightarrow{QS} = 2\langle -1, -12 \rangle$

$= \boxed{\langle \underset{v_1}{-2}, \underset{v_2}{-24} \rangle}$

Magnitude: $|2\overrightarrow{QS}| = \sqrt{(-2)^2 + (-24)^2}$

$= \sqrt{4 + 576}$

$= \sqrt{580}$

$= \sqrt{4} \cdot \sqrt{145}$

$= \boxed{2\sqrt{145}}$

(11) $3\overrightarrow{QR} + \overrightarrow{PS}$ $\quad Q = (\overset{x_1}{3}, \overset{y_1}{4})$, $R = (\overset{x_2}{-2}, \overset{y_2}{5})$

$\underbrace{P = (-2, 2)}_{\text{Initial pt}}$, $\underbrace{S = (2, -8)}_{\text{terminal pt.}}$

Component form: $\overrightarrow{QR} = \langle -5, 1 \rangle$ (see #7 above)

$\overrightarrow{PS} = \langle 2-(-2), -8-2 \rangle$

$= \langle 4, -10 \rangle$

$\therefore 3\overrightarrow{QR} + \overrightarrow{PS} = 3\langle -5, 1 \rangle + \langle 4, -10 \rangle$

$= \langle -15, 3 \rangle + \langle 4, -10 \rangle$

$= \langle -15+4, 3-10 \rangle$

$= \boxed{\langle \underset{v_1}{-11}, \underset{v_2}{-7} \rangle}$

Magnitude: $|3\overrightarrow{QR} + \overrightarrow{PS}| = \sqrt{(-11)^2 + (-7)^2}$

$= \sqrt{121 + 49}$

$= \boxed{\sqrt{170}}$

(13) $\vec{u} + \vec{v} = \langle -1, 3 \rangle + \langle 2, 4 \rangle$

$= \langle -1+2, 3+4 \rangle$

$= \boxed{\langle 1, 7 \rangle}$

(15) $\vec{u} - \vec{w} = \langle -1, 3 \rangle - \langle 2, -5 \rangle$

$= \langle -1, 3 \rangle + \langle -2, 5 \rangle$

$= \langle -1-2, 3+5 \rangle$

$= \boxed{\langle -3, 8 \rangle}$

(17) $2\vec{u} + 3\vec{w} = 2\langle -1, 3 \rangle + 3\langle 2, -5 \rangle$

$= \langle -2, 6 \rangle + \langle 6, -15 \rangle$

$= \langle -2+6, 6-15 \rangle$

$= \boxed{\langle 4, -9 \rangle}$

(19) $-2\vec{u} - 3\vec{v} = -2\langle -1, 3 \rangle - 3\langle 2, 4 \rangle$

$= \langle 2, -6 \rangle + \langle -6, -12 \rangle$

$= \langle 2-6, -6-12 \rangle$

$= \boxed{\langle -4, -18 \rangle}$

PA8-8 p. 464 #21-39 odd

(21) Given: $\vec{v} = \langle -2, 4 \rangle$ (u_1, u_2)

Recall unit vector: $\vec{u} = \left\langle \frac{v_1}{|\vec{v}|}, \frac{v_2}{|\vec{v}|} \right\rangle$

$|\vec{v}| = \sqrt{(-2)^2+4^2} = \sqrt{4+16} = \sqrt{20} = \sqrt{4}\cdot\sqrt{5}$

$\therefore, |\vec{v}| = 2\sqrt{5}$

$\therefore$, Unit vector (in direction of $\vec{v}$):

$\left\langle \frac{-2}{2\sqrt{5}}, \frac{4}{2\sqrt{5}} \right\rangle = \boxed{\left\langle -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle}$

(23) $\vec{w} = -\vec{i} - 2\vec{j} = \langle -1, -2 \rangle$

$|\vec{w}| = \sqrt{(-1)^2+(-2)^2} = \sqrt{1+4} = \sqrt{5}$

$\therefore$, Unit vector (in the direction of $\vec{w}$):

$\vec{u} = \left\langle \frac{-1}{\sqrt{5}}, \frac{-2}{\sqrt{5}} \right\rangle = \boxed{-\frac{1}{\sqrt{5}}\vec{i} - \frac{2}{\sqrt{5}}\vec{j}}$

(25) $\vec{u} = \langle 2, 1 \rangle$

$|\vec{u}| = \sqrt{2^2+1^2} = \sqrt{5}$

$\therefore$, Unit vector (in direction of $\vec{u}$):

a) component form: $\boxed{\left\langle \frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle}$

b) Linear combination of vectors $\vec{i}$ and $\vec{j}$: $\boxed{\frac{2}{\sqrt{5}}\vec{i} + \frac{1}{\sqrt{5}}\vec{j}}$

(27) $\vec{u} = \langle -4, -5 \rangle$

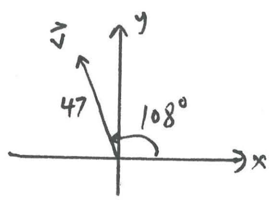
$|\vec{u}| = \sqrt{(-4)^2+(-5)^2} = \sqrt{16+25} = \sqrt{41}$

$\therefore$, Unit vector (in direction of $\vec{u}$):

a) Component form: $\boxed{\left\langle -\frac{4}{\sqrt{41}}, -\frac{5}{\sqrt{41}} \right\rangle}$

b) Linear combination of vectors $\vec{i}$ and $\vec{j}$: $\boxed{-\frac{4}{\sqrt{41}}\vec{i} - \frac{5}{\sqrt{41}}\vec{j}}$

(31)

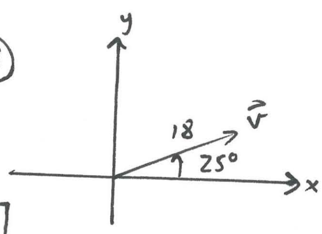


$\vec{v} = \langle |\vec{v}|\cdot\cos\theta, |\vec{v}|\sin\theta \rangle$

$= \langle 47\cdot\cos 108°, 47\cdot\sin 108° \rangle$

$\boxed{= \langle -14.524, 44.700 \rangle}$

(29)

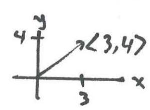


$\vec{v} = \langle |\vec{v}|\cos\theta, |\vec{v}|\sin\theta \rangle$

$= \langle 18\cdot\cos 25°, 18\cdot\sin 25° \rangle$

$\boxed{= \langle 16.314, 7.607 \rangle}$

(33) $\langle 3, 4 \rangle$ Quadrant I



Magnitude: $|\langle 3, 4 \rangle| = \sqrt{3^2+4^2}$

$= \sqrt{9+16}$

$= \sqrt{25}$

$\boxed{= 5}$

Recall: $\vec{v} = \langle a, b \rangle = \langle |\vec{v}|\cos\theta, |\vec{v}|\sin\theta \rangle$

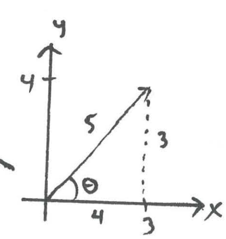
$\therefore, \langle 3, 4 \rangle = \langle 5\cos\theta, 5\sin\theta \rangle$

① $3 = 5\cos\theta$ ② $4 = 5\sin\theta$

$\frac{3}{5} = \cos\theta$ $\frac{4}{5} = \sin\theta$

$\cos^{-1}\left(\frac{3}{5}\right) = \theta$ $\sin^{-1}\left(\frac{4}{5}\right) = \theta$

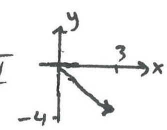
$53.130° = \theta$ $53.130° = \theta$



$\boxed{\therefore, \text{Direction } \angle \text{ of } \langle 3,4 \rangle \text{ is } 53.130°}$

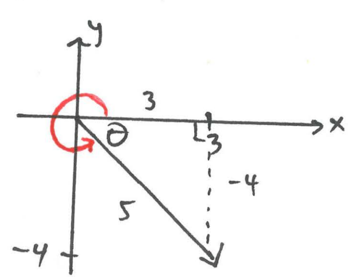
Algebraic method

(35) $3i - 4j = \langle 3, -4 \rangle$ Quad IV



Magnitude: $|\langle 3, -4 \rangle| = \sqrt{3^2+(-4)^2} = \sqrt{25} = 5$

Graphical approch:



Use any trig ratio to find θ:

I'll use sine since the domain is restricted on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] = [-90°, 90°]$.

$\sin\theta = \frac{-4}{5} \Rightarrow \theta = \sin^{-1}\left(-\frac{4}{5}\right)$

$= -53.130°$

Direction ∠

$360 - 53.130 = \boxed{306.870°}$

(37) $7(\cos 135^\circ \vec{i} + \sin 135^\circ \vec{j})$ linear comb. form

$= 7\cos 135^\circ \vec{i} + 7\sin 135^\circ \vec{j}$

$= \langle 7\cos 135^\circ, 7\sin 135^\circ \rangle$ Component form

$\boxed{\therefore \text{, Magnitude of } 7(\cos 135^\circ \vec{i} + \sin 135^\circ \vec{j}) \text{ is } 7.}$

$\boxed{\therefore \text{, Direction } \angle \text{ of } 7(\cos 135^\circ i + \sin 135^\circ j) \text{ is } 135^\circ.}$

(39) Given: $|\vec{v}| = 2$, $\vec{u} = \langle 3, -3 \rangle$

Need: vector $\vec{v}$ in the same direction as $\vec{u}$.

Step 1: Find the unit vector

$|\vec{u}| = \sqrt{3^2 + (-3)^2} = \sqrt{9+9} = \sqrt{9 \cdot 2} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$

$\therefore$, Unit vector (in direction of $\vec{v}$): $\langle \frac{3}{3\sqrt{2}}, \frac{-3}{3\sqrt{2}} \rangle = \langle \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \rangle$

Step 2: Multiply $\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \rangle$ by the magnitude of vector $\vec{v}$, which is 2 (given)

$\vec{v} = 2\langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \rangle = \langle \frac{2}{\sqrt{2}}, \frac{-2}{\sqrt{2}} \rangle = \langle \frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}, \frac{-2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \rangle$

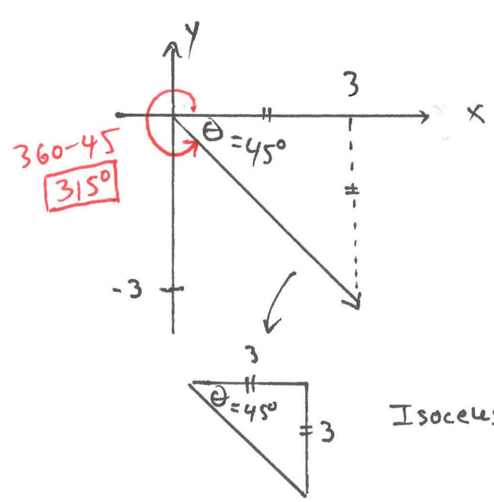
$= \langle \frac{2\sqrt{2}}{\sqrt{4}}, \frac{-2\sqrt{2}}{\sqrt{4}} \rangle$

$= \langle \frac{\cancel{2}\sqrt{2}}{\cancel{2}}, -\frac{\cancel{2}\sqrt{2}}{\cancel{2}} \rangle$

$= \boxed{\langle \sqrt{2}, -\sqrt{2} \rangle}$

OR

Sketch vector $\vec{u}$ and do a little right triangle trig.



Isoceles right triangle

$\vec{v} = \langle |\vec{v}|\cos\theta, |\vec{v}|\sin\theta \rangle$

$= \langle 2 \cdot \cos 315^\circ, 2 \cdot \sin 315^\circ \rangle$

$= \langle 2 \cdot \frac{\sqrt{2}}{2}, 2(-\frac{\sqrt{2}}{2}) \rangle$

$= \boxed{\langle \sqrt{2}, -\sqrt{2} \rangle}$

$|\vec{v}| = 2$

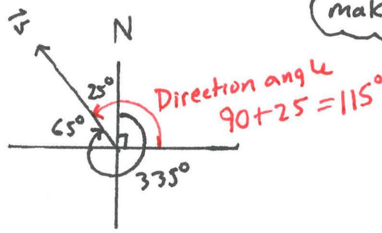
Unit circle
$\cos 315^\circ = \frac{\sqrt{2}}{2}$
$\sin 315^\circ = -\frac{\sqrt{2}}{2}$

P8-9 p. 465 # 41–51 odd

Bearing – angle the line of travel makes due North, measured clockwise

(41) Bearing 335°

$|\vec{v}| = 530$ mph



$\vec{v} = \langle |\vec{v}|\cdot\cos\theta,\ |\vec{v}|\sin\theta\rangle$

$= \langle 530\cdot\cos 115°,\ 530\sin 115°\rangle$

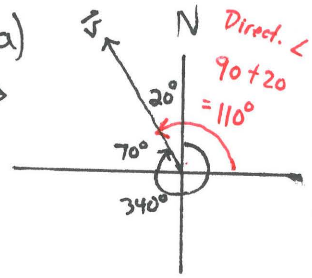
$\boxed{\vec{v} = \langle -223.988,\ 480.343\rangle}$

(43) Bearing of airplane 340°

$|\vec{v}| = 325$ mph (speed) → a)

Bearing of wind 320°

$|\vec{w}| = 40$ mph

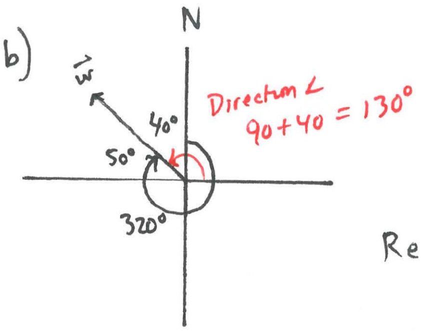


$\vec{v} = \langle |\vec{v}|\cos\theta,\ |\vec{v}|\sin\theta\rangle$

$= \langle 325\cdot\cos 110°,\ 325\sin 110°\rangle$

$\boxed{\vec{v} = \langle -111.157,\ 305.400\rangle}$

b)



$\vec{w} = \langle |\vec{w}|\cos\theta,\ |\vec{w}|\sin\theta\rangle$

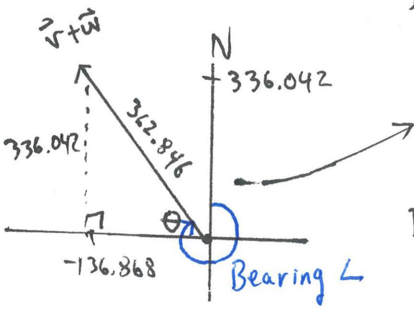
$= \langle 40\cos 130°,\ 40\sin 130°\rangle$

$\vec{w} = \langle -25.711,\ 30.642\rangle$

Resulting velocity: $\vec{v}+\vec{w} = \langle -111.157 + (-25.711),\ 305.400 + 30.642\rangle$

$= \langle -136.868,\ 336.042\rangle$

∴, Ground speed of plane: $|\vec{v}+\vec{w}| = \sqrt{(-136.868)^2 + 336.042^2} = \boxed{362.846\text{mph}}$



$\tan\theta = \dfrac{336.042}{-136.868} \Rightarrow \theta = \tan^{-1}\left(\dfrac{336.042}{-136.868}\right) = -67.839°$

Thus, $|\theta| = 67.839°$.

Direction of plane: $270 + 67.839 = \boxed{337.839°}$

(45) Basketball shot at 70° w/ horizontal direction

Initial speed 10 m/sec. $|\vec{v}| = 10$ m/sec

a) Component form (of initial velocity): $\vec{v} = \langle |\vec{v}|\cos\theta,\ |\vec{v}|\sin\theta\rangle$

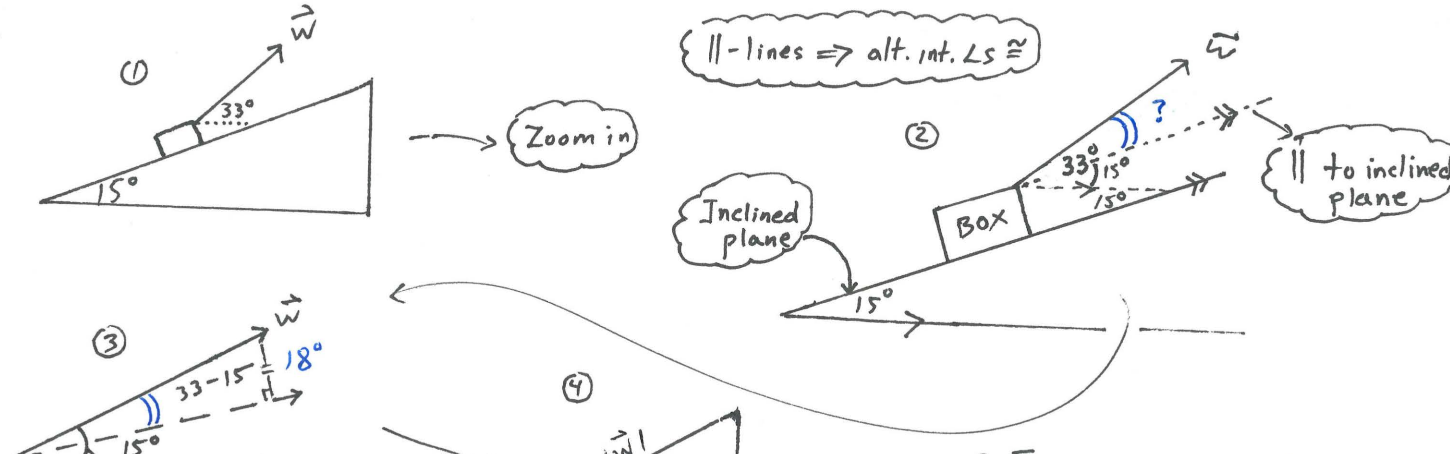
$= \langle 10\cdot\cos 70°,\ 10\cdot\sin 70°\rangle$

$\boxed{\vec{v} = \langle 3.420,\ 9.397\rangle}$

b) The ball's horizontal speed is 3.420 meters per second.

The ball's vertical speed is 9.397 meters per second.

(47)



$\cos 18° = \dfrac{2.5}{|\vec{w}|} \Rightarrow |\vec{w}| \cdot \cos 18° = 2.5$

$\therefore,\ |\vec{w}| = \dfrac{2.5}{\cos 18°} = 2.629$

$\therefore,\ \vec{w} = \langle |\vec{w}| \cdot \cos\theta,\ |\vec{w}| \cdot \sin\theta \rangle$

$= \langle 2.629 \cdot \cos 33°,\ 2.629 \cdot \sin 33° \rangle$

$\boxed{\vec{w} = \langle 2.205,\ 1.432 \rangle}$

(49) Combining forces (Addition of force vectors)

Vectors: $\vec{F}_1 = \langle |\vec{F}_1| \cos\theta,\ |\vec{F}_1| \sin\theta \rangle$

$= \langle 50 \cdot \cos 45°,\ 50 \cdot \sin 45° \rangle$

$= \langle 50 \cdot \frac{\sqrt{2}}{2},\ 50 \cdot \frac{\sqrt{2}}{2} \rangle$ Unit ⊙

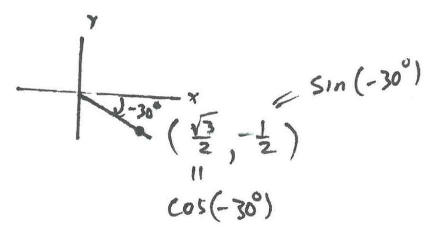
$= \langle 25\sqrt{2},\ 25\sqrt{2} \rangle$

$\vec{F}_2 = \langle |\vec{F}_2| \cos\theta,\ |\vec{F}_2| \sin\theta \rangle$

$= \langle 75 \cdot \cos(-30°),\ 75 \cdot \sin(-30°) \rangle$

$= \langle 75 \cdot \frac{\sqrt{3}}{2},\ 75(-\frac{1}{2}) \rangle$

$= \langle \frac{75\sqrt{3}}{2},\ -\frac{75}{2} \rangle$



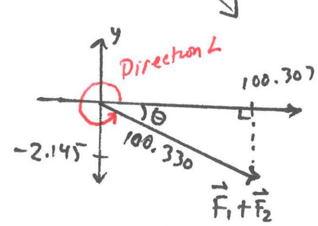
$\therefore$ Direction of combined forces: $\vec{F}_1 + \vec{F}_2 = \langle 25\sqrt{2},\ 25\sqrt{2} \rangle + \langle \frac{75\sqrt{3}}{2},\ -\frac{75}{2} \rangle$

$= \langle 25\sqrt{2} + \frac{75\sqrt{3}}{2},\ 25\sqrt{2} + (-\frac{75}{2}) \rangle$

$= \langle 100.307,\ -2.145 \rangle$

Finally, magnitude: $|\vec{F}_1 + \vec{F}_2| = \sqrt{(100.307)^2 + (-2.145)^2}$

$\boxed{= 100.330 \text{ lbs}}$



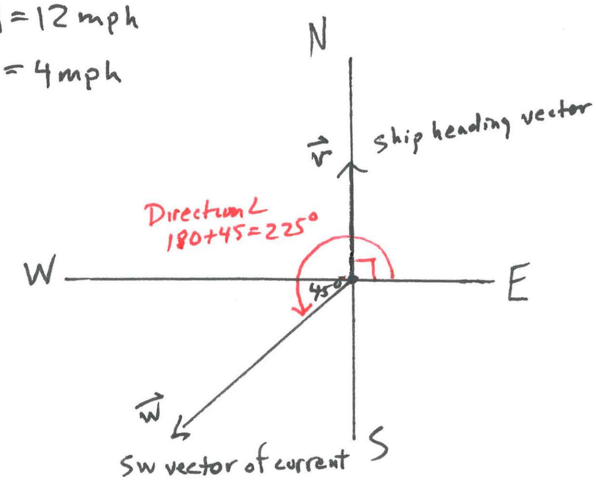
$\theta = \sin^{-1}\left(\frac{-2.145}{100.330}\right) = -1.225°$

$360 - 1.225 = \boxed{358.775°}$

(51) Ship heading due N @ 12 mph , $|\vec{v}| = 12$ mph

Current flowing SW @ 4 mph , $|\vec{w}| = 4$ mph

Find actual bearing and speed of ship



Ship heading vector:

$$\vec{v} = \langle |\vec{v}|\cos\theta , |\vec{v}|\sin\theta \rangle$$

$$= \langle 12 \cdot \cos 90°, 12 \sin 90° \rangle$$

$$= \langle 12 \cdot 0 , 12 \cdot 1 \rangle \leftarrow \begin{array}{l} \cos 90° = 0 \\ \sin 90° = 1 \end{array}$$

$$\vec{w} = \langle 0, 12 \rangle$$

Current flow:

$$\vec{w} = \langle 4\cos 225°, 4\sin 225° \rangle$$

$$= \langle 4\left(-\tfrac{\sqrt{2}}{2}\right), 4\left(-\tfrac{\sqrt{2}}{2}\right) \rangle \leftarrow \text{Again, unit ☺}$$

$$= \langle -2\sqrt{2}, -2\sqrt{2} \rangle$$

∴, Ship's velocity vector: $\vec{v} + \vec{w} = \langle 0, 12 \rangle + \langle -2\sqrt{2}, -2\sqrt{2} \rangle$

$$= \langle 0 - 2\sqrt{2} , 12 - 2\sqrt{2} \rangle$$

$$= \langle -2\sqrt{2} , 12 - 2\sqrt{2} \rangle$$

$$= \langle -2.828, 9.172 \rangle$$

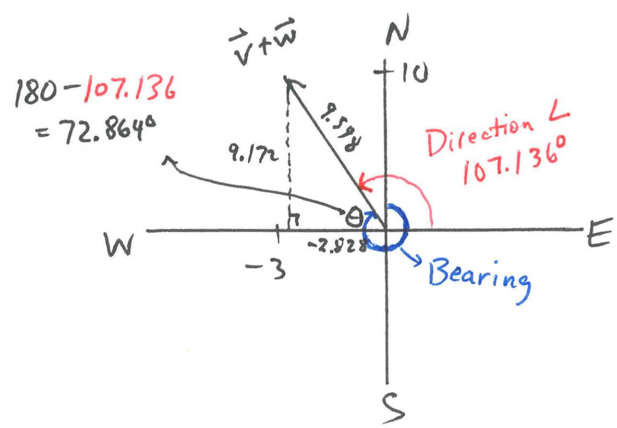
∴, Speed of ship: $|\vec{v} + \vec{w}| = \sqrt{(-2.828)^2 + 9.172^2} = \boxed{9.598 \text{ mph}}$

$$\cos\theta' = \frac{-2.828}{9.598}$$

$$\theta' = \cos^{-1}\left(\frac{-2.828}{9.598}\right) = 107.136°$$

∴, Ships bearing: $270 + 72.864$

$$= \boxed{342.864°}$$



PA 8-10 p. 472 # 1-23 odd

$\vec{u}\cdot\vec{v} = \langle a,b\rangle\cdot\langle c,d\rangle = ac + bd$

① $\vec{u}\cdot\vec{v} = \langle 5,3\rangle\cdot\langle 12,4\rangle = 5(12) + 3(4) = 60 + 12 = \boxed{72}$

③ $\vec{u}\cdot\vec{v} = \langle 4,5\rangle\cdot\langle -3,-7\rangle = 4(-3) + 5(-7) = -12 - 35 = \boxed{-47}$

⑤ $\vec{u} = -4\vec{i} - 9\vec{j} = \langle -4,-9\rangle$

$\vec{v} = -3\vec{i} - 2\vec{j} = \langle -3,-2\rangle$

$\therefore,\ \vec{u}\cdot\vec{v} = \langle -4,-9\rangle\cdot\langle -3,-2\rangle = (-4)(-3) + (-9)(-2) = 12 + 18 = \boxed{30}$

⑦ $\vec{u} = 7\vec{i} = 7\vec{i} + 0\vec{j} = \langle 7,0\rangle$

$\vec{v} = -2\vec{i} + 5\vec{j} = \langle -2,5\rangle$

$\therefore,\ \vec{u}\cdot\vec{v} = \langle 7,0\rangle\cdot\langle -2,5\rangle = 7(-2) + 0(5) = -14 + 0 = \boxed{-14}$

⑨ Dot product property # 2 (p. 467) : $|\vec{u}|^2 = \vec{u}\cdot\vec{u} \Rightarrow |\vec{u}| = \sqrt{\vec{u}\cdot\vec{u}}$

$\vec{u}\cdot\vec{u} = \langle 5,-12\rangle\cdot\langle 5,-12\rangle = 5(5) + (-12)(-12) = 25 + 144 = 169$

$\therefore,\ |\vec{u}| = \sqrt{\vec{u}\cdot\vec{u}} = \sqrt{169} = \boxed{13}$

⑪ $\vec{u} = -4\vec{i} = -4\vec{i} + 0\vec{j} = \langle -4,0\rangle$

$\vec{u}\cdot\vec{u} = \langle -4,0\rangle\cdot\langle -4,0\rangle = (-4)(-4) + 0\cdot 0 = 16 + 0 = 16$

$\therefore,\ |\vec{u}| = \sqrt{\vec{u}\cdot\vec{v}} = \sqrt{16} = \boxed{4}$

⑬ $\vec{u} = \langle -4,-3\rangle\ ,\ \vec{v} = \langle -1,5\rangle$

$\vec{u}\cdot\vec{v} = \langle -4,-3\rangle\cdot\langle -1,5\rangle = (-4)(-1) + (-3)(5) = 4 - 15 = -11$

$|\vec{u}| = \sqrt{(-4)^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$

$|\vec{v}| = \sqrt{(-1)^2 + 5^2} = \sqrt{1+25} = \sqrt{26}$

$\therefore,\ \cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \frac{-11}{5\sqrt{26}} \Rightarrow \theta = \cos^{-1}\left(\frac{-11}{5\sqrt{26}}\right) = \boxed{115.560°}$

⑮ $\vec{u} = \langle 2,3\rangle\ ,\ \vec{v} = \langle -3,5\rangle$

$\vec{u}\cdot\vec{v} = \langle 2,3\rangle\cdot\langle -3,5\rangle = 2(-3) + 3(5) = -6 + 15 = 9$

$|\vec{u}| = \sqrt{2^2 + 3^2} = \sqrt{4+9} = \sqrt{13}$

$|\vec{v}| = \sqrt{(-3)^2 + 5^2} = \sqrt{9+25} = \sqrt{34}$

$\therefore,\ \cos\theta = \frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \frac{9}{\sqrt{13}\cdot\sqrt{34}} \Rightarrow \theta = \cos^{-1}\left(\frac{9}{\sqrt{13}\cdot\sqrt{34}}\right) = \boxed{64.654°}$

(17) $\vec{u} = 3\vec{i} - 3\vec{j} = \langle 3,-3 \rangle$

$\vec{v} = -2\vec{i} + 2\sqrt{3}\vec{j} = \langle -2, 2\sqrt{3} \rangle$

$\vec{u}\cdot\vec{v} = \langle 3,-3\rangle\cdot\langle -2,2\sqrt{3}\rangle = 3(-2) + (-3)(2\sqrt{3}) = -6 - 6\sqrt{3}$

$|\vec{u}| = \sqrt{3^2+(-3)^2} = \sqrt{9+9} = \sqrt{9\cdot 2} = 3\sqrt{2}$

$|\vec{v}| = \sqrt{(-2)^2+(2\sqrt{3})^2} = \sqrt{4+2^2(\sqrt{3})^2} = \sqrt{4+4(3)} = \sqrt{4+12} = \sqrt{16} = 4$

$\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \dfrac{-6-6\sqrt{3}}{(3\sqrt{2})(4)} = \dfrac{-6(1+\sqrt{3})}{12\sqrt{2}} = -\dfrac{1+\sqrt{3}}{2\sqrt{2}}$

$\therefore$, $\cos\theta = -\dfrac{1+\sqrt{3}}{2\sqrt{2}} \Rightarrow \theta = \cos^{-1}\left(-\dfrac{1+\sqrt{3}}{2\sqrt{2}}\right) = \boxed{165^\circ}$

(19) $\vec{u} = \left(2\cos\frac{\pi}{4}\right)\vec{i} + \left(2\sin\frac{\pi}{4}\right)\vec{j} = 2\left(\frac{\sqrt{2}}{2}\right)\vec{i} + 2\left(\frac{\sqrt{2}}{2}\right)\vec{j} = \sqrt{2}\vec{i} + \sqrt{2}\vec{j} = \langle \sqrt{2},\sqrt{2}\rangle$

$\vec{v} = \left(\cos\frac{3\pi}{2}\right)\vec{i} + \left(\sin\frac{3\pi}{2}\right)\vec{j} = 0\vec{i} - \vec{j} = \langle 0,-1\rangle$

$\vec{u}\cdot\vec{v} = \langle\sqrt{2},\sqrt{2}\rangle\cdot\langle 0,-1\rangle = \sqrt{2}(0) + \sqrt{2}(-1) = 0 - \sqrt{2} = -\sqrt{2}$

$|\vec{u}| = \sqrt{(\sqrt{2})^2+(\sqrt{2})^2} = \sqrt{2+2} = \sqrt{4} = 2$

$|\vec{v}| = \sqrt{0^2+(-1)^2} = \sqrt{1} = 1$

$\therefore$, $\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \dfrac{-\sqrt{2}}{(2)(1)} = -\dfrac{\sqrt{2}}{2} \Rightarrow \theta = \cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) \Rightarrow \boxed{\theta = \frac{3\pi}{4} \text{ or } 135^\circ}$

(21) From diagram, $\vec{u} = \langle 8,5\rangle$ and $\vec{v} = \langle -3,4\rangle$.

$\vec{u}\cdot\vec{v} = \langle 8,5\rangle\cdot\langle -3,4\rangle = 8(-3) + 5(4) = -24 + 20 = -4$

$|\vec{u}| = \sqrt{8^2+5^2} = \sqrt{64+25} = \sqrt{89}$

$|\vec{v}| = \sqrt{(-3)^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$

$\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \dfrac{-4}{5\sqrt{89}} \Rightarrow \theta = \cos^{-1}\left(\dfrac{-4}{5\sqrt{89}}\right) = \boxed{94.865^\circ}$

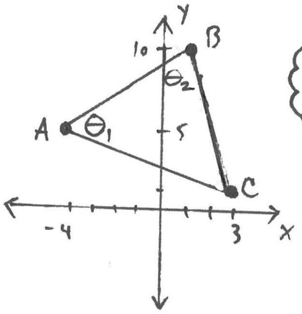
(23) Recall: $\vec{u}\cdot\vec{v} = 0 \iff \vec{u},\vec{v}$ orthogonal

Pf: $\vec{u}\cdot\vec{v} = \langle 2,3\rangle\cdot\langle \frac{3}{2},-1\rangle = 2\left(\frac{3}{2}\right) + 3(-1) = 3 - 3 = 0$

$\therefore$, Since $\vec{u}\cdot\vec{v} = 0$, $\vec{u},\vec{v}$ are ortlogonal :)

PA 8-11 p. 473 #29-43 odd

(29) A $(-4,5)$, B $(1,10)$, C $(3,1)$



Remember Head-Tail

Let's solve using vectors. Start by choosing the angles you want to find, say $\angle A$ and $\angle B$. Now, construct the vectors between angles.

$\overrightarrow{AB} = \langle 1-(-4), 10-5 \rangle = \langle 5,5 \rangle = \vec{u}$

$\overrightarrow{AC} = \langle 3-(-4), 1-5 \rangle = \langle 7,-4 \rangle = \vec{v}$

$\vec{u}\cdot\vec{v} = \langle 5,5 \rangle \cdot \langle 7,-4 \rangle = 5(7)+5(-4) = 35-20 = 15$

$|\vec{u}| = \sqrt{5^2+5^2} = \sqrt{25+25} = \sqrt{25\cdot 2} = 5\sqrt{2}$

$|\vec{v}| = \sqrt{7^2+(-4)^2} = \sqrt{49+16} = \sqrt{65}$

$\therefore, \cos\theta_1 = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \dfrac{15}{(5\sqrt{2})(\sqrt{65})} \Rightarrow \boxed{\theta_1 = \angle A = 74.745^\circ}$

$\overrightarrow{BA} = \langle -4-1, 5-10 \rangle = \langle -5,-5 \rangle = \vec{u}$, $|\vec{u}| = \sqrt{(-5)^2+(-5)^2} = \sqrt{25+25} = \sqrt{25.2} = 5\sqrt{2}$

$\overrightarrow{BC} = \langle 3-1, 1-10 \rangle = \langle 2,-9 \rangle = \vec{v}$, $|\vec{v}| = \sqrt{2^2+(-9)^2} = \sqrt{4+81} = \sqrt{85}$

$\vec{u}\cdot\vec{v} = \langle -5,-5 \rangle \cdot \langle 2,-9 \rangle = (-5)(2)+(-5)(-9) = -10+45 = 35$

$\therefore, \cos\theta_2 = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} = \dfrac{35}{(5\sqrt{2})(\sqrt{85})} \Rightarrow \boxed{\theta_2 = \angle B = 57.529^\circ}$

$\therefore, \angle C = 180 - \angle A - \angle B = 180^\circ - 74.745^\circ - 57.529^\circ = \boxed{47.726^\circ}$

(31) Given: $\theta = 150^\circ$, $|\vec{u}| = 3$, $|\vec{v}| = 8$

$\cos\theta = \dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|} \Rightarrow \cos 150^\circ = \dfrac{\vec{u}\cdot\vec{v}}{(3)(8)} = \dfrac{\vec{u}\cdot\vec{v}}{24} \Rightarrow 24\cdot\cos 150^\circ = \vec{u}\cdot\vec{v}$

$24\left(-\frac{\sqrt{3}}{2}\right) = \vec{u}\cdot\vec{v}$

$\boxed{\therefore, -12\sqrt{3} = \vec{u}\cdot\vec{v}}$

(33) $\vec{u} = \langle 5,3 \rangle$, $\vec{v} = \langle -\frac{10}{4}, -\frac{3}{2} \rangle$

$\vec{u} \parallel \vec{v}$ if slopes $=$ (parallel)

Slope $m = \dfrac{\text{RISE}}{\text{RUN}} = \dfrac{\Delta Y}{\Delta X}$

$\vec{u},\vec{v}$ orthogonal if $\vec{u}\cdot\vec{v} = 0$ (Dot product $=0$)

$m_{\vec{u}} = \dfrac{3}{5}$, $m_{\vec{v}} = \dfrac{-\frac{3}{2}}{-\frac{10}{4}} = \dfrac{3}{2}\cdot\dfrac{4}{10} = \dfrac{12}{20} = \dfrac{3}{5}$

Since the slope of $\vec{u}$ and $\vec{v}$ is the same

$\Rightarrow \boxed{\vec{u} \parallel \vec{v}}$ (Parallel)

(35) $\vec{u} = \langle 15,-12 \rangle$, $\vec{v} = \langle -4,5 \rangle$

$m_{\vec{u}} = \dfrac{-12}{15} = -\dfrac{4}{5}$, $m_{\vec{v}} = \dfrac{5}{-4}$

$\therefore, m_{\vec{u}} \nparallel m_{\vec{v}}$ (NOT Parallel)

$\vec{u}\cdot\vec{v} = \langle 15,-12 \rangle \cdot \langle -4,5 \rangle$
$= 15(-4) + (-12)(5)$
$= -60 - 60$
$= -120$
$\neq 0$

$\therefore, \vec{u},\vec{v}$ NOT orthogonal

$\therefore$, Since $m_{\vec{u}} \nparallel m_{\vec{v}}$ and $\vec{u},\vec{v}$ NOT orthogonal

$\boxed{\therefore, \text{Neither}}$

(37) $\vec{u}\cdot\vec{v} = \langle -3,4\rangle \cdot \langle 20,15\rangle = -3(20)+4(15) = -60+60 = 0$

$\boxed{\therefore,\ \vec{u},\vec{v} \text{ are orthogonal}}$

(39) a) Set $y=0$ to find x-intercept A

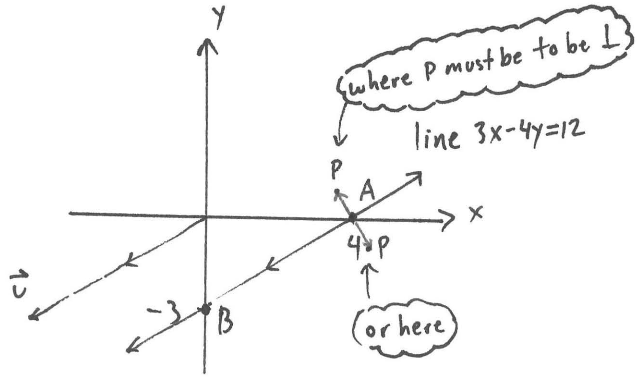
$3x-4y=12$
$3x-4(0)=12$
$3x=12$
$x=4$

$\boxed{\therefore,\ \text{x-intercept } A: (4,0)}$

Set $x=0$ to find y-intercept B

$3x-4y=12$
$3(0)-4y=12$
$-4y=12$
$y=-3$

$\boxed{\therefore,\ \text{y-intercept } B: (0,-3)}$



b) Need to find coordinates of point $P(x,y)$ such that $\vec{AP} \perp$ line and $|\vec{AP}|=1$. (magnitude must be 1)

First, find component form of $\vec{AB}$.

$\vec{AB} = \vec{u} = \langle 0-4, -3-0\rangle = \langle -4,-3\rangle$

Notice $\vec{u} \parallel$ line $3x-4y=12$.

Since $\vec{u}=\langle -4,-3\rangle$, the direction of a vector $\perp$ to $\vec{u}$ must be $\underbrace{\langle 3,-4\rangle}_{\vec{v}}$ or $\underbrace{\langle -3,4\rangle}_{\vec{w}}$.

Recall lines are $\perp$ when slopes are opposite reciprocals of each other.

Slope of $\vec{u}$: $\frac{-3}{-4}=\frac{3}{4}$; slope of $\vec{v}=\langle 3,-4\rangle$: $\frac{-4}{3}$; slope of $\vec{w}=\langle -3,4\rangle$: $\frac{4}{-3}$. Also, notice

$$\left.\begin{aligned}\vec{u}\cdot\vec{v} &= \langle -4,-3\rangle\langle 3,-4\rangle = (-4)(3)+(-3)(-4) = -12+12=0\\ \vec{u}\cdot\vec{w} &= \langle -4,-3\rangle\langle -3,4\rangle = (-4)(-3)+(-3)(4) = 12-12=0\end{aligned}\right\}\text{ vectors orthogonal } \checkmark$$

$\therefore,\ |\vec{AP}|=1$ if $\vec{AP}=\frac{\vec{v}}{|\vec{v}|} = \frac{\langle 3,-4\rangle}{\sqrt{3^2+(-4)^2}} = \frac{\langle 3,-4\rangle}{\sqrt{25}} = \frac{\langle 3,-4\rangle}{5} = \langle \frac{3}{5}, -\frac{4}{5}\rangle$ and

$|\vec{AP}|=1$ if $\vec{AP}=\frac{\vec{w}}{|\vec{w}|} = \frac{\langle -3,4\rangle}{\sqrt{(-3)^2+4^2}} = \frac{\langle -3,4\rangle}{\sqrt{25}} = \frac{\langle -3,4\rangle}{5} = \langle -\frac{3}{5}, \frac{4}{5}\rangle$

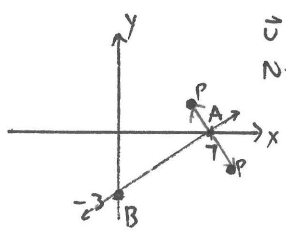
$\boxed{\therefore,\ \text{Point P must be: } (4+\tfrac{3}{5}, -\tfrac{4}{5}) = (4.6, -0.8) \text{ and } (4-\tfrac{3}{5}, \tfrac{4}{5}) = (3.4, 0.8)}$

(41) a) $3x-7y=21$
$3x-7(0)=21$
$3x=21$
$x=7$

$\boxed{\text{x-intercept } A: (7,0)}$

$3x-7y=21$
$3(0)-7y=21$
$-7y=21$
$y=-3$

$\boxed{\text{x-intercept } B: (0,-3)}$



$\vec{u}=\vec{AB}=\langle 0-7,-3-0\rangle = \langle -7,-3\rangle$

Need vector $\vec{v}=\langle 3,-7\rangle$ or $\vec{w}=\langle -3,7\rangle$ so that $\vec{AP}\perp 3x-7y=21$

$|\vec{AP}|=1$ if $\vec{AP}=\frac{\vec{v}}{|\vec{v}|} = \frac{\langle 3,-7\rangle}{\sqrt{3^2+(-7)^2}} = \frac{\langle 3,-7\rangle}{\sqrt{58}} = \langle \frac{3}{\sqrt{58}}, \frac{-7}{\sqrt{58}}\rangle$

$|\vec{AP}|=1$ if $\vec{AP}=\frac{\vec{w}}{|\vec{w}|} = \frac{\langle -3,7\rangle}{\sqrt{58}} = \langle -\frac{3}{\sqrt{58}}, \frac{7}{\sqrt{58}}\rangle$

$\boxed{\therefore,\ \text{Point P must be: } (7+\tfrac{3}{\sqrt{58}}, \tfrac{-7}{\sqrt{58}}) \text{ or } (7-\tfrac{3}{\sqrt{58}}, \tfrac{7}{\sqrt{58}})}$

(43) Find vectors satisfying $\vec{u} = \langle 2,3\rangle$, $\vec{u}\cdot\vec{v} = 10$, and $|\vec{v}|^2 = 17$

Let $\vec{v} = \langle x, y\rangle$.

$\vec{u}\cdot\vec{v} = 10 \Rightarrow \langle 2,3\rangle \cdot \langle x,y\rangle = 10$ (Dot product)

$\Rightarrow 2x + 3y = 10$

$|\vec{v}|^2 = 17 \Rightarrow |\langle x,y\rangle|^2 = 17$

$\Rightarrow \left(\sqrt{x^2+y^2}\right)^2 = 17$ (Magnitude: $|\vec{v}| = \sqrt{v_1^2+v_2^2}$)

$\Rightarrow x^2 + y^2 = 17$

Now, solve $2x+3y=10$ for x (or y) and substitute into $x^2+y^2=17$.

$2x + 3y = 10$

$2x = 10 - 3y$

$x = \frac{10}{2} - \frac{3}{2}y$

$x = \underline{5 - \frac{3}{2}y}$

$x^2 + y^2 = 17$

$(5 - \frac{3}{2}y)^2 + y^2 = 17$

$(5-\frac{3}{2}y)(5-\frac{3}{2}y) + y^2 = 17$ (Distribute)

$25 - \underbrace{\frac{15}{2}y - \frac{15}{2}y} + \underbrace{\frac{9}{4}y^2 + y^2} = 17$

$25 - \frac{30}{2}y + \frac{9}{4}y^2 + y^2 = 17$ (Like terms)

$25 - 15y + \frac{9}{4}y^2 + \frac{4}{4}y^2 = 17$ (common denominator)

$25 - 15y + \frac{13}{4}y^2 = 17$

$-17 \qquad\qquad -17$

$8 - 15y + \frac{13}{4}y^2 = 0$ (Rearrange)

$\frac{13}{4}y^2 - 15y + 8 = 0$ (Multiply eq by 4 to eliminate denomin.)

$13y^2 - 60y + 32 = 0$

Apply QF ; $y = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ← $a = 13$, $b = -16$, $c = 32$

$= \dfrac{-(-60) \pm \sqrt{(-60)^2 - 4(13)(32)}}{2(13)}$

$= \dfrac{60 \pm \sqrt{3600 - 1664}}{26}$

$= \dfrac{60 \pm \sqrt{1936}}{26}$

$= \dfrac{60 \pm 44}{26}$

$\therefore, y_1 = \dfrac{60+44}{26} = \dfrac{104}{26} = 4$

$y_2 = \dfrac{60-44}{26} = \dfrac{16}{26} = \dfrac{8}{13}$

If $y_1 = 4$, then $x = 5 - \frac{3}{2}(4) = 5 - 6 = -1$

$\boxed{\therefore, \langle -1, 4\rangle}$

If $y_2 = \frac{8}{13}$, then $x = 5 - \frac{3}{2}\left(\frac{8}{13}\right)$

$= 5 - \frac{12}{13}$

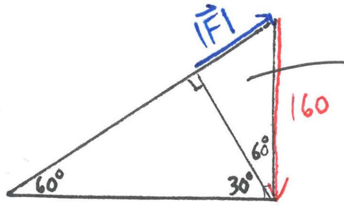
$= \frac{65}{13} - \frac{12}{13}$

$= \frac{53}{13}$

$\boxed{\therefore, \langle \frac{53}{13}, \frac{8}{13}\rangle}$

P8-12 p.473 # 45 – 55 odd

(45)



$\sin 60° = \dfrac{|\vec{F}|}{160}$

$160 \cdot \sin 60° = |\vec{F}|$

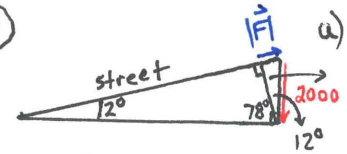
$160 \cdot \dfrac{\sqrt{3}}{2} = |\vec{F}|$

$80\sqrt{3} = |\vec{F}|$

$\boxed{138.564 \text{ lbs} = |\vec{F}|}$

The force required for Mandisa to keep the sled from sliding down the hill is 138.564 lbs

(47)



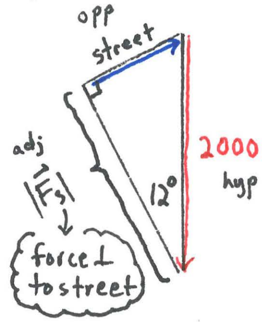
a) $\sin 12° = \dfrac{|\vec{F}|}{2000}$

$2000 \cdot \sin 12° = |\vec{F}|$

$\boxed{415.823 \text{ lbs} = \vec{F}}$

The force required to keep car from rolling down the hill is 415.823 lbs.

b) Force ⊥ to street?



$\cos 12° = \dfrac{|\vec{F_s}|}{2000}$

$2000 \cdot \cos 12° = |\vec{F_s}|$

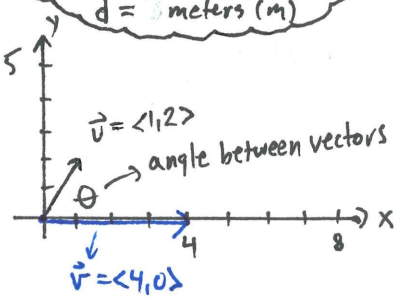
$\boxed{1956.295 \text{ lbs} = |\vec{F_s}|}$

(49) $W = F \cdot d$

$= (2600 \text{ lbs})(5.5 \text{ ft})$

$\boxed{\therefore,\ W = 14{,}300 \text{ ft-lbs}}$

(51) $|\vec{F}| = 12$ Newtons (N)
d = meters (m)



$\vec{F} = \langle \underbrace{|\vec{F}|\cos\theta}_{\text{Horiz. force}},\ \underbrace{|\vec{F}|\sin\theta}_{\text{Vertical force}} \rangle$

$\therefore,\ W = (\text{Horizontal force}) \cdot \text{distance}$

$= |\vec{F}| \cos\theta \cdot d$

$= (12 \text{ N})\left(\dfrac{1}{\sqrt{5}}\right)(4 \text{ m})$

$= \dfrac{48}{\sqrt{5}}$ N-m

$\boxed{\therefore,\ W = 21.466 \text{ Joules}}$

$\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|} = \dfrac{\cancel{4}}{\cancel{4}\sqrt{5}} = \dfrac{1}{\sqrt{5}}$

$\vec{u} \cdot \vec{v} = \langle 1,2 \rangle \cdot \langle 4,0 \rangle$

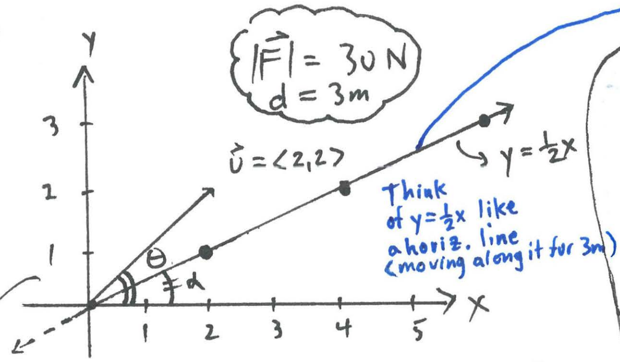
$= 1(4) + 2(0)$

$= 4$

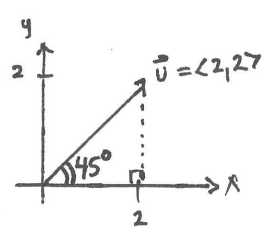
$|\vec{u}| = \sqrt{1^2 + 2^2} = \sqrt{1+4} = \sqrt{5}$

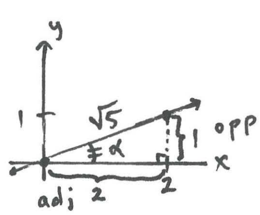
$|\vec{v}| = \sqrt{4^2 + 0^2} = \sqrt{16} = 4$

(53) $|\vec{F}| = 30$ N
$d = 3$m



line $y = \frac{1}{2}x$ slope $= m = \dfrac{1}{2}\ \dfrac{\text{RISE}}{\text{RUN}}$





$1^2 + 2^2 = c^2$

$5 = c^2$

$\sqrt{5} = c$

$\cos\alpha = \dfrac{2}{\sqrt{5}}$

$\alpha = \cos^{-1}\left(\dfrac{2}{\sqrt{5}}\right) = 26.565°$

$\therefore,\ \theta = 45° - \alpha$

$\theta = 45° - 26.565°$

$\theta = 18.435°$

$W = |\vec{F}| \cos\theta \cdot d$

$= (30\text{N})(\cos 18.435°)(3\text{m})$

$= 85.381$ N-m

$\boxed{\therefore,\ W = 85.381 \text{ Joules}}$

(55) $|\vec{F}| = 200$ lbs

$\vec{AB} = 2\vec{i} + 3\vec{j} = \langle 2,3 \rangle$ Moving along this vector (from A to B)

The angle between is 30°.

$d = ?$ Yes. $|\vec{AB}| = |\langle 2,3 \rangle| = \sqrt{2^2 + 3^2} = \sqrt{13} = d$

$\text{Work} = \vec{F} \cdot d$

$= |\vec{F}| \cos\theta \cdot d$

$= (200 \text{ lbs})(\cos 30°)(\sqrt{13} \text{ ft})$

$= 200\left(\dfrac{\sqrt{3}}{2}\right)\sqrt{13}$ ft-lbs

$= 100\sqrt{39}$ ft-lbs

$\boxed{\therefore,\ W \approx 624.5 \text{ ft-lbs}}$:)