

Advanced Algebra

Unit 8 Practice: Rational Expression and Functions

$$f(x) = \frac{5}{(x-3)(x+4)}$$

Apply

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Objective

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Check for Understanding

Concept Check

1. Sample answer:

$$\frac{4}{6} \cdot \frac{4(x+2)}{6(x+2)}$$

1. OPEN ENDED Write two rational expressions that are equivalent.
2. Explain how multiplication and division of rational expressions are similar to multiplication and division of rational numbers. **See margin.**
3. Determine whether $\frac{2d+5}{3d+5} = \frac{2}{3}$ is sometimes, always, or never true. Explain. **Never; solving the equation using cross products leads to $15 = 10$, which is never true.**

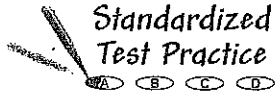
Guided Practice

Simplify each expression.

GUIDED PRACTICE KEY

Exercises	Examples
4-6	1, 3
7-10	4-6
11, 12	7
13	2

4. $\frac{45mn^3}{20n^7} \cdot \frac{9m^2}{4n^4}$
5. $\frac{a+b}{a^2-b^2} \cdot \frac{1}{a-b}$
6. $\frac{6y^3-9y^2}{2y^2+5y-12} \cdot \frac{3y^4}{y+4}$
7. $\frac{2a^2}{5b^2c} \cdot \frac{3bc^2}{8a^2} \cdot \frac{3c}{20b}$
8. $\frac{35}{16x^2} \div \frac{21}{4x} \cdot \frac{5}{12x}$
9. $\frac{3t+6}{7t-7} \cdot \frac{14t-14}{5t+10}$
10. $\frac{12p^2+6p-6}{4(p+1)^2} \div \frac{6p-3}{2p+10}$
11. $\frac{\frac{c^3d^3}{a}}{\frac{xc^2d}{ax^2}} \cdot cd^2x$
12. $\frac{\frac{2y}{y^2-4}}{\frac{3}{y^2-4y+4}} \cdot \frac{2y(y-2)}{3(y+2)}$



Standardized Test Practice

13. Identify all of the values of y for which the expression $\frac{y-4}{y^2-4y-12}$ is undefined. **D**
 (A) -2, 4, 6 (B) -6, -4, 2 (C) -2, 0, 6 (D) -2, 6

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
14-21	1, 3
22-35	4-6
36-41	7
42, 43, 50	2

Extra Practice

See page 847.

P12
Choose:
14-34 even
-OR-
15-35 odd

Simplify each expression.

14. $\frac{30bc}{12b^2} \cdot \frac{5c}{2b}$
15. $\frac{-3mn^4}{21m^2n^2} \cdot \frac{-n^2}{7m}$
16. $\frac{(-3x^2y)^3}{9x^2y^2} - 3x^4y$
17. $\frac{(-2rs^2)^2}{12r^2s^3} \cdot \frac{s}{3}$
18. $\frac{5t-5}{t^2-1} \cdot \frac{5}{t+1}$
19. $\frac{c+5}{2c+10} \cdot \frac{1}{2}$
20. $\frac{y^2+4y+4}{3y^2+5y-2} \cdot \frac{y+2}{3y-1}$
21. $\frac{a^2+2a+1}{2a^2+3a+1} \cdot \frac{a+1}{2a+1}$
22. $\frac{3xyz}{4xz} \cdot \frac{6x^2}{3y^2} \cdot \frac{3x^2}{2y}$
23. $\frac{-4ab}{21c} \cdot \frac{14c^2}{18a^2} \cdot \frac{4bc}{27a}$
24. $\frac{3}{5d} \div \left(\frac{-9}{15df} \right) - f$
25. $\frac{p^3}{2q} \div \frac{-p}{4q} - 2p^2$
26. $\frac{2x^3y}{z^5} \div \left(\frac{4xy}{z^3} \right)^2 \cdot \frac{xz}{8y}$
27. $\frac{xy}{a^3} \div \left(\frac{xy}{ab} \right)^3 \cdot \frac{b^3}{x^2y^2}$
28. $\frac{3t^2}{t+2} \cdot \frac{t+2}{t^2} \cdot 3$
29. $\frac{4w+4}{3} \cdot \frac{1}{w+1} \cdot \frac{4}{3}$
30. $\frac{4t^2-4}{9(t+1)^2} \cdot \frac{3t+3}{2t-2} \cdot \frac{2}{3}$
31. $\frac{3p-21}{p^2-49} \cdot \frac{p^2+7p}{3p} \cdot 1$
32. $\frac{5(x-3)}{2(x+1)}$
33. $\frac{w^2-11w+24}{w^2-18w+80} \cdot \frac{w^2-15w+50}{w^2-9w+20} \cdot \frac{w-3}{w-4}$
34. $\frac{r^2+2r-8}{r^2+4r+3} \div \frac{r-2}{3r+3} \cdot \frac{3(r+4)}{r+3}$
35. $\frac{a^2+2a-15}{a-3} \div \frac{a^2-4}{2} \cdot \frac{2(a+5)}{(a-2)(a+2)}$

DAILY INTERVENTION

Differentiated Instruction

Intrapersonal Have students think about what aspects of multiplying and dividing rational expressions they find most challenging. Have them



Basketball
After graduating from U.S. Naval Academy, Robinson became a rookie of the year. He has played in different Olympics.
Source: NBA

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
14-21	1, 2
22-39	3, 4
40-43	5
44-49	6

Extra Practice

P13

CHOOSE:

22-36 even

OR

23-37 odd

Find the LCM of each set of polynomials.

14. $10s^2, 35s^2t^2, 70s^2t^2$

16. $14a^3, 15bc^3, 12b^3, 420a^3b^3c^3$

18. $4w - 12, 2w - 6, 4(w - 3)$

20. $2t^2 + t - 3, 2t^2 + 5t + 3$
 $(2t + 3)(t - 1)(t + 1)$

Simplify each expression.

22. $\frac{6}{ab} + \frac{8}{a} \frac{5 + 8b}{ab}$

24. $\frac{5}{r} + 7 \frac{5 + 7r}{r}$

26. $\frac{3x}{4y^2} - \frac{y}{6x} \frac{9x^2 - 2y^3}{12xy^2}$

28. $\frac{3}{4q} - \frac{2}{5q} - \frac{1}{2q} - \frac{3}{20q}$

30. $\frac{7}{y - 8} - \frac{6}{8 - y} \frac{13}{y - 8}$

32. $\frac{m}{m^2 - 4} + \frac{2}{3m + 6} \frac{5m - 4}{3(m + 2)(m - 2)}$

15. $36x^2y, 20xyz, 180x^2yz$

17. $9p^2q^3, 6pq^4, 4p^3, 36p^3q^4$

19. $x^2 - y^2, x^3 + x^2y, x^2(x - y)(x + y)$

21. $n^2 - 7n + 12, n^2 - 2n - 8$
 $(n - 4)(n - 3)(n + 2)$

23. $\frac{5}{6v} + \frac{7}{4v} \frac{31}{12v}$

25. $\frac{2x}{3y} + 5 \frac{2x + 15y}{3y}$

27. $\frac{5}{a^2b} - \frac{7a}{5b^2} \frac{25b - 7a^3}{5a^2b^2}$

29. $\frac{11}{9} - \frac{7}{2w} - \frac{6}{5w} \frac{110w - 423}{90w}$

31. $\frac{a}{a - 4} - \frac{3}{4 - a} \frac{a + 3}{a - 4}$

33. $\frac{y}{y + 3} - \frac{6y}{y^2 - 9} \frac{y(y - 9)}{(y + 3)(y - 3)}$

35. $\frac{-8d + 20}{(d - 4)(d + 4)(d - 2)}$

36. $\frac{-4h + 15}{(h - 4)(h - 5)^2}$

37. $\frac{x^2 - 6}{(x + 2)^2(x + 3)}$

39. $\frac{2y^2 + y - 4}{(y - 1)(y - 2)}$

34. $\frac{5}{x^2 - 3x - 28} + \frac{7}{2x - 14} \frac{7x + 38}{2(x - 7)(x + 4)}$

36. $\frac{1}{h^2 - 9h + 20} - \frac{5}{h^2 - 10h + 25}$

★ 38. $\frac{m^2 + n^2}{m^2 - n^2} + \frac{m}{n - m} + \frac{n}{m + n} 0$

★ 40. $\frac{\frac{1}{b + 2} + \frac{1}{b - 5}}{\frac{2b^2 - b - 3}{b^2 - 3b - 10}} \frac{1}{b + 1}$

35. $\frac{d - 4}{d^2 + 2d - 8} - \frac{d + 2}{d^2 - 16}$

37. $\frac{x}{x^2 + 5x + 6} - \frac{2}{x^2 + 4x + 4}$

39. $\frac{y + 1}{y - 1} + \frac{y + 2}{y - 2} + \frac{y}{y^2 - 3y + 2}$

41. $\frac{(x + y)\left(\frac{1}{x} - \frac{1}{y}\right)}{(x - y)\left(\frac{1}{x} + \frac{1}{y}\right)} - 1$

Practice and Apply

Homework Help

For Exercises	See Examples
16–21	1
22–39	2, 3
40–50	4

Extra Practice

See page 849.

asymptote:

$x = -1$; hole: $x = 5$

p14 - #16 - 2/11

Determine the equations of any vertical asymptotes and the values of x for any holes in the graph of each rational function.

16. $f(x) = \frac{2}{x^2 - 5x + 6}$ asymptotes: $x = 2, x = 3$
17. $f(x) = \frac{4}{x^2 + 2x - 8}$ asymptotes: $x = -4, x = 2$
18. $f(x) = \frac{x + 3}{x^2 + 7x + 12}$ asymptote: $x = -4$; hole: $x = -3$
19. $f(x) = \frac{x - 5}{x^2 - 4x - 5}$
20. $f(x) = \frac{x^2 - 8x + 16}{x - 4}$ hole: $x = 4$
21. $f(x) = \frac{x^2 - 3x + 2}{x - 1}$ hole: $x = 1$

Graph each rational function. 22–39. See pp. 519A–519D.

22. $f(x) = \frac{1}{x}$
23. $f(x) = \frac{3}{x}$
24. $f(x) = \frac{1}{x + 2}$
25. $f(x) = \frac{-5}{x + 1}$
26. $f(x) = \frac{x}{x - 3}$
27. $f(x) = \frac{5x}{x + 1}$
28. $f(x) = \frac{-3}{(x - 2)^2}$
29. $f(x) = \frac{1}{(x + 3)^2}$
30. $f(x) = \frac{x + 4}{x - 1}$
31. $f(x) = \frac{x - 1}{x - 3}$
32. $f(x) = \frac{x^2 - 36}{x + 6}$
33. $f(x) = \frac{x^2 - 1}{x - 1}$
34. $f(x) = \frac{3}{(x - 1)(x + 5)}$
35. $f(x) = \frac{-1}{(x + 2)(x - 3)}$
36. $f(x) = \frac{x}{x^2 - 1}$
37. $f(x) = \frac{x - 1}{x^2 - 4}$
38. $f(x) = \frac{6}{(x - 6)^2}$
39. $f(x) = \frac{1}{(x + 2)^2}$

More About...



lory •
 Mathematician Maria
 Agnesi was one of
 the greatest scholars of all
 times.

• **HISTORY** For Exercises 40–42, use the following information.

In Maria Gaetana Agnesi's book *Analytical Institutions*, Agnesi discussed the characteristics of the equation $x^2y = a^2(a - y)$, whose graph is called the "curve of Agnesi." This equation can be expressed as $y = \frac{a^3}{x^2 + a^2}$.

40. Graph $f(x) = \frac{a^3}{x^2 + a^2}$ if $a = 4$. See pp. 519A–519D.

41. Describe the graph.

42. Make a conjecture about the shape of the graph of $f(x) = \frac{a^3}{x^2 + a^2}$ if $a = -4$. Explain your reasoning. See pp. 519A–519D.

41. The graph is bell-shaped with a horizontal asymptote at $f(x) = 0$.

AUTO SAFETY For Exercises 43–45, use the following information.

When a car has a front-end collision, the objects in the car (including passengers) keep moving forward until the impact occurs. After impact, objects are repelled. Seat belts and airbags limit how far you are jolted forward. The formula for the velocity at which you are thrown backward is $V_f = \frac{(m_1 - m_2)v_i}{m_1 + m_2}$, where m_1 and m_2 are masses

Target 8E

Date _____ Period _____

Solve each equation. Remember to check for extraneous solutions.

1) $\frac{3x+3}{x^2} - \frac{1}{x^2} = \frac{5}{x}$

2) $\frac{1}{2r^2} = \frac{5}{2r^2} + \frac{1}{2r}$

3) $\frac{1}{n} - \frac{n+1}{5n} = \frac{3}{n}$

4) $\frac{x-5}{4x^2} + \frac{1}{x} = \frac{x+3}{x^2}$

5) $1 - \frac{n+1}{n+6} = \frac{4n-24}{n+6}$

6) $\frac{2}{b^2+4b-5} + \frac{1}{b-1} = \frac{3}{b^2+4b-5}$

7) $\frac{5r}{r+4} + \frac{r-1}{r+4} = 2$

8) $\frac{1}{x} = \frac{x+2}{x^2+x} - \frac{1}{x+1}$

9) $\frac{1}{p} = \frac{1}{p^2} + \frac{p+3}{p}$

10) $\frac{6}{k^2} = \frac{1}{5k} + \frac{1}{5}$

11) $\frac{b^2+2b-24}{b+1} + \frac{1}{b+1} = b-1$

12) $\frac{n+5}{n+6} - \frac{1}{n^2+9n+18} = \frac{2}{n^2+9n+18}$

Answers to Target 8E

1) $\{1\}$

2) $\{-4\}$

3) $\{-11\}$

4) $\{17\}$

5) $\left\{\frac{29}{4}\right\}$

6) $\{-4\}$

7) $\left\{\frac{9}{4}\right\}$

8) $\{1\}$

9) $\{-1\}$

10) $\{-6, 5\}$

11) $\{11\}$

12) $\{-2\}$

★ indicates increased difficulty

Practice and Apply

Homework Help

For Exercises	See Examples
14–37	1–3
38–53	4

Extra Practice
See page 848.

State whether each equation represents a *direct*, *joint*, or *inverse* variation. Then name the constant of variation. 14. direct; 1.5 15. $a = 5bc$ joint; 5 16. $vw = -18$ inverse; -18

14. $\frac{n}{m} = 1.5$

15. $a = 5bc$ joint; 5

16. $vw = -18$

17. $3 = \frac{a}{b}$ direct; 3

18. $p = \frac{12}{q}$

inverse; 12

19. $y = -7x$

direct; -7

20. $V = \frac{1}{3}Bh$

joint; $\frac{1}{3}$

21. $\frac{2.5}{t} = s$

inverse; 2.5

22. **CHEMISTRY** Boyle's Law states that when a sample of gas is kept at a constant temperature, the volume varies inversely with the pressure exerted on it. Write an equation for Boyle's Law that expresses the variation in volume V as a function of pressure P . $V = \frac{k}{P}$

23. **CHEMISTRY** Charles' Law states that when a sample of gas is kept at a constant pressure, its volume V will increase as the temperature t increases. Write an equation for Charles' Law that expresses volume as a function. $V = kt$

24. **GEOMETRY** How does the circumference of a circle vary with respect to its radius? What is the constant of variation? directly; 2π

25. **TRAVEL** A map is scaled so that 3 centimeters represents 45 kilometers. How far apart are two towns if they are 7.9 centimeters apart on the map? 118.5 km

Find each value.

26. If y varies directly as x and $y = 15$ when $x = 3$, find y when $x = 12$. 60

27. If y varies directly as x and $y = 8$ when $x = 6$, find y when $x = 15$. 20

28. Suppose y varies jointly as x and z . Find y when $x = 2$ and $z = 27$, if $y = 192$ when $x = 8$ and $z = 6$. 216

29. If y varies jointly as x and z and $y = 80$ when $x = 5$ and $z = 8$, find y when $x = 16$ and $z = 2$. 64

30. If y varies inversely as x and $y = 5$ when $x = 10$, find y when $x = 2$. 25

31. If y varies inversely as x and $y = 16$ when $x = 5$, find y when $x = 20$. 4

32. If y varies inversely as x and $y = 2$ when $x = 25$, find x when $y = 40$. 1.25

33. If y varies inversely as x and $y = 4$ when $x = 12$, find y when $x = 5$. 9.6

34. If y varies directly as x and $y = 9$ when $x = -15$, find y when $x = 21$. -12.6

35. If y varies directly as x and $x = 6$ when $y = 0.5$, find y when $x = 10$. 0.83

★ 36. Suppose y varies jointly as x and z . Find y when $x = \frac{1}{2}$ and $z = 6$, if $y = 45$ when $x = 6$ and $z = 10$. $2\frac{1}{2}$

★ 37. If y varies jointly as x and z and $y = \frac{4}{8}$ when $x = \frac{1}{2}$ and $z = 3$, find y when $x = 6$ and $z = \frac{1}{3}$. $\frac{1}{6}$

38. **WORK** Paul drove from his house to work at an average speed of 40 miles per hour. The drive took him 15 minutes. If the drive home took him 20 minutes and he used the same route in reverse, what was his average speed going home? 30 mph

39. **WATER SUPPLY** Many areas of Northern California depend on the snowpack of the Sierra Nevada Mountains for their water supply. If 250 cubic centimeters of snow will melt to 28 cubic centimeters of water, how much water does 900 cubic centimeters of snow produce? 100.8 cm^3

Career Choices



Travel Agent

Travel agents give advice and make arrangements for transportation, accommodations, and recreation. For international travel, they also provide information on customs and currency exchange.



Online Research

For information about a career as a travel agent, visit: www.algebra2.com/careers

Advanced Algebra
Unit 8 PRACTICE SWIG

Name: P17
Hour: _____ Date: _____

DIRECTIONS: You must show your work to receive full credit.

8.A. Classify an equation as direct, inverse or joint variation

1. State whether each equation represents *direct*, *joint*, or *inverse* variation. Then state k , the constant of variation.

a) $y = 5xz$ b) $v = 4w$ c) $xy = \frac{1}{3}$ d) $y = \frac{7}{x}$ e) $0.5 = \frac{m}{n}$

8.B. Create equations to solve direct, inverse or joint variation problems.

2. The variable y varies directly with x . If $y = -16$ when $x = 4$, find x when $y = 24$.

3. If y varies directly as x and $y = 18$ when $x = 15$, find y when $x = 20$.

4. The variable y varies inversely with x . If $y = 2$ when $x = 8$, find x when $y = 32$.

5. If y varies inversely as x and $y = -14$ when $x = 12$, find x when $y = 21$.

6. The variable y varies jointly with x and z . If $y = 24$ when $z = 2$ and $x = 1$, find y when $x = 12$ and $z = 2$.

7. Suppose y varies jointly as x and z . Find y when $x = 9$ and $z = -5$, if $y = -90$ when $z = 15$ and $x = -6$.

8.C. Perform operations and simplify rational expressions.

Simplify each expression.

$$8. \frac{x^6}{x-8} \cdot \frac{x^2-8x}{x^3}$$

$$9. \frac{x+y}{8} \div \frac{x^2-y^2}{4}$$

$$10. \frac{x^2+2x-3}{x^2+4x+4} \cdot \frac{x^2-4}{x^2-3x+2}$$

$$11. \frac{3x+6}{40x^2} \div \frac{3x^2}{24}$$

$$12. \frac{x-3}{x-1} + \frac{6}{5x-5}$$

$$13. \frac{3}{2x} - \frac{1}{6x} - \frac{2}{4x}$$

$$14. \frac{6}{d^2+4d+4} + \frac{5}{d+2}$$

8.D. Understand the relationship between a rational function and its graph.

15. For each function, determine the equations of any vertical asymptotes and the values of x for any holes in the graph.

a) $f(x) = \frac{3}{(x-5)(x+2)}$

b) $f(x) = \frac{x^2-9}{x+3}$

8.E. Solve rational equations.

16. Solve each equation. Check for extraneous solutions.

a) $\frac{4}{x-1} = 2 - \frac{x}{x-1}$

b) $\frac{4}{x-2} = \frac{1}{x-3}$