Name: _____

Probability Unit Review

1. Decide if each demonstrates independent or dependent probability.
   A. Landing on heads after tossing a coin and rolling a 2 on a single 6-sided die. I
   B. Choosing a marble from a jar and landing on tails after tossing a coin. I
   C. Choosing a 9 from a deck of cards, replacing it, and then choosing a jack as the second card. I
   D. A card is chosen at random from a standard deck of 52 playing cards, and without replacing it, a second card is chosen. D
   E. Rolling a 2 on a single 6-sided die, and then rolling a 3 on a second roll of the die. I
   F. Choosing a blue candy from a dish, eating it, and then choosing a red one. D

2. Today's forecast calls for a 30% chance of rain in Chicago and a 58% chance of rain in London. The probability that it will rain in both cities is 17.4%. Are the events "It rains in London" and "It rains in Chicago" dependent or independent? Justify your answer by demonstrating/explaining a probability test.

   Test: $P(A \cap B) \stackrel{?}{=} P(A) \cdot P(B)$
   $0.174 \stackrel{?}{=} 0.30 \cdot 0.58$
   $0.174 = 0.174$ ✓
   Since $P(A \cap B) = P(A) \cdot P(B)$, the events are independent (as we expect).

3. The probability that you will graduate college is 0.60. The probability that you will have a job making lots of money is 0.48. The probability that you will graduate college and earn lots of money is 0.50. Are these events dependent or independent? Justify your answer by demonstrating/explaining a probability test.

   Test: $P(A \cap B) \stackrel{?}{=} P(A) \cdot P(B)$
   $0.50 \stackrel{?}{=} 0.60 \cdot 0.48$
   $0.50 \neq 0.288$
   So $P(A \cap B) \neq P(A) \cdot P(B)$
   ∴ The events are dependent.

4. Mr. McMahon and Morty the Mustang are playing a game where they flip a fair coin four times and try to predict the outcomes. Using the sample space of possible outcomes listed below, answer each of the following questions.

~ HHHH	~HHHT	TTHT	HTTH
THHH	TTTT	TTTH	TTHH
~ HTHH	HTTT	HHTT	THTH
HHTH	THTT	~HTHT	THHT

- a) What is P(A), the probability that the third flip is heads? $8/16 = \frac{1}{2}$
- b) What is P(B), the probability that the first flip is tails? $8/16 = \frac{1}{2}$
- c) What is P(A and B), the probability that the third flip is heads *and* the first flip is heads? $4/16 = \frac{1}{4}$
- d) Are events A and B independent? $P(A \cap B) \stackrel{?}{=} P(A) P(B) \Rightarrow \frac{1}{4} = \frac{1}{2} \cdot \frac{1}{2} \Rightarrow \frac{1}{4} = \frac{1}{4}$ ✓ So yes, indep.
- e) Find the P(all four heads) $= \frac{1}{16}$
- f) Find the probability that you flipped all four and any two ~~where~~ were heads? $\frac{6}{16} = \frac{3}{8}$
- g) Was the sample space provided necessary to do this problem? Was it useful for some and not others? Explain. You answer this question

Total $7+3+4+6=20$

5. A goblet contains 7 red marbles, 3 green marbles, 4 yellow, and 6 blue marbles.

a. If we choose a marble, then another marble *without putting the first one back in the goblet*, what is the probability that the first marble will be blue and the second will be red?

Depend $P(A \cap B) = \frac{6}{20} \cdot \frac{7}{19} = \boxed{\frac{21}{190} \approx 11\%}$ (A = blue, B = red)

$\frac{6}{20} \cdot \frac{7}{19} = \frac{42}{380}$

b. If we choose three marbles, with replacement, what is the probability that all are green?

Indept $P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C) = \frac{3}{20} \cdot \frac{3}{20} \cdot \frac{3}{20} = \frac{27}{8000} \approx \boxed{0.003375}$ or $\boxed{0.3375\%}$

c. What is the probability of first choosing two red in a row, then two green in a row without replacement.

Depend $P(A \cap B \cap C \cap D) = \frac{7}{20} \cdot \frac{6}{19} \cdot \frac{3}{18} \cdot \frac{2}{17} = \frac{7}{3230} \approx \boxed{0.002}$ or $\boxed{0.2\%}$

6. The compound probability of two independent events is found by which operation: $P(A) \cdot P(B)$

a. Adding b. Subtracting c. Multiplying d. Dividing e.Crying

7. Find $P(A$ and $B)$ if A and B are independent events, $P(A) = 0.58$ and $P(B) = 0.26$

$P(A \text{ and } B) = P(A) \cdot P(B) = 0.58 \cdot 0.26 = \boxed{0.1508}$

8. What is the probability of rolling a 4, or a number less than 2 on a regular die? (A = rolling a 4, B = less than 2)

$P(A \cup B) = P(A) + P(B) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \boxed{\frac{1}{3}}$

9. What is the probability of rolling a 4 and then a number less than 2 on a regular die?

$P(A \cap B) = P(A) \cdot P(B) = \frac{1}{6} \cdot \frac{1}{6} = \boxed{\frac{1}{36}}$

10. Sara wins 5 out of every 6 math competitions. Jose loses 3 out of every 8 dancing competitions that he enters. What is the probability that Sara and Jose both win their next events?

Sara wins: $\frac{5}{6}$ Jose loses $\frac{3}{8}$ → this means he wins $1 - \frac{3}{8} = \frac{8}{8} - \frac{3}{8} = \frac{5}{8}$

$\therefore P(\text{Sara wins} \cap \text{Jose wins}) = \frac{5}{6} \cdot \frac{5}{8} = \boxed{\frac{25}{48}}$

11. John wins 2 out of 3 baseball games. Damien wins 3 out of 5 baseball games. What is the probability they both win their next game?

$\frac{2}{3} \cdot \frac{3}{5} = \frac{6}{15} = \boxed{\frac{2}{5}}$

12. In your drawer you have 5 shirts colored white, brown, green, pink, and blue, and 4 pairs of pants colored blue, yellow, orange, and violet. You grab a shirt and pants at random.

a. What is the probability that you select either a brown shirt and either blue or orange pants?

$\frac{1}{5} \cdot \left(\frac{1}{4} + \frac{1}{4}\right) = \frac{1}{5} \cdot \frac{2}{4} = \frac{2}{20} = \boxed{\frac{1}{10}}$

b. What is the probability that you select a white shirt and violet pants? (A = white shirt, B = violet pants)

$P(A \cap B) = \frac{1}{5} \cdot \frac{1}{4} = \boxed{\frac{1}{20}}$

13. The probability of eating pizza tonight is 0.38 and the probability of seeing a movie is 0.23. If the probability of eating pizza *given* that you see a movie is 0.38, is eating pizza independent of seeing a movie? Show work to support your answer (using the conditional probability formula) (A = eating pizza, B = seeing a movie)

Use formula to test for independence:

Is $P(A|B) = P(A)$?

✓ $0.38 = 0.38$

Yes, eating pizza is independent of seeing movie since $P(A|B) = P(A)$.

↳ must show this work!

14. The probability that your parents give you $40 for the weekend (A) is 0.25 and the probability that you will go to a concert on Saturday night (B) is 0.60. If the probability of your parents giving you money (A) *given* you attend the concert (B) is 0.48, are the events independent? Show work to support your answer(use the conditional probability formula)

Does $P(A|B) = P(A)$? If so, independent. Otherwise NOT, independent.

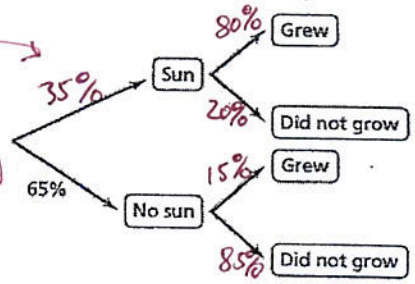
$0.48 \neq 0.25$ ∴ Not independent.

15. A school survey found that 7 out of 9 students like Twix candy. If three students are chosen at random with replacement, what is the probability that all three students like Twix?

$P(\text{all 3 like Twix}) = \frac{7}{9} \cdot \frac{7}{9} \cdot \frac{7}{9} \approx 0.471$

16. In a science class you grew plants with and without sun.
65% of plants got no sun. 80% of the plants with sun grew. 15% of the plants with no sun grew.

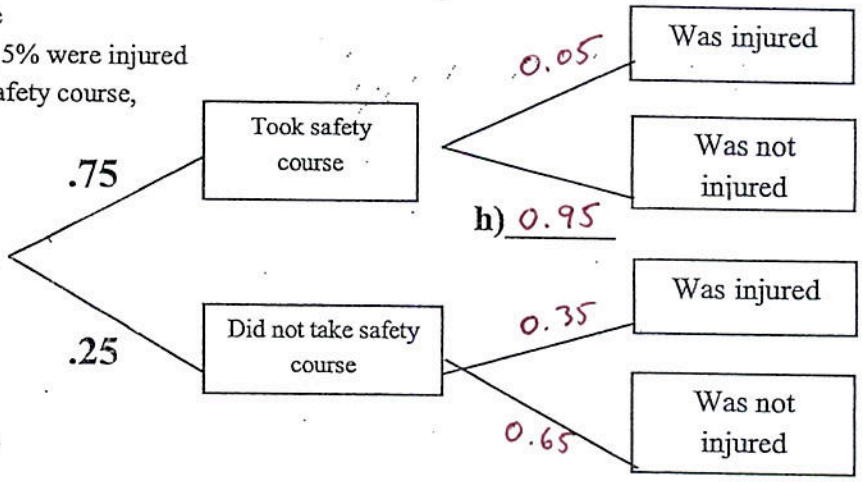
a. Fill in the tree with all missing percentages
b. What is the overall percentage of plants that grew?
c. Find P(no sun *and* grew). ⇒ $P(\text{no sun}) \cdot P(\text{Grew}) = 0.65 \cdot 0.15 = 0.0975$ or 9.75%
d. Find P(grew | sun) = 0.80



b) Overall = $P(\text{Grew}|\text{Sun})P(\text{Sun}) + P(\text{Grew}|\text{No Sun})P(\text{No Sun})$
$= 0.80 \cdot 0.35 + 0.15 \cdot 0.65$
$= 0.3775$
$= 37.75\%$

d)

17. A survey reported the following data on students who enrolled in a wood working class and were injured.
- 75% of students took the safety course
- Of the students who took safety class, 5% were injured
- Of the students who did not take the safety course, 35% were injured



a. On the tree diagram, which decimal should be placed in the blank space labeled "h"?
b. Explain in words what the probability of **h** represents.
The prob. of a student not injured given student (he or she) took safety course.
c. Find $P(\textit{took safety class AND was not injured}) = 0.75 \cdot 0.95 = 0.7125$ or 71.25%

d. What is the overall percentage of students who got injured.

Overal % = $P(\text{Injured}|\text{took course}) \cdot P(\text{took course}) + P(\text{Injured}|\text{did not take}) \cdot P(\text{did not take})$
$= 0.05 \cdot 0.75 + 0.35 \cdot 0.25$
$= 0.125$ or 12.5%

18. What is the probability that a randomly chosen animal is a dog that lives inside?

7/40

19. What is P(is a cat | lives outside)?

8/25

	Lives outside	Live inside	Totals
Dogs	17	7	24
Cats	8	8	16
Totals	25	15	40

20. Use the frequency table to consider events A and B below. Are the events independent? Justify your reasoning.

Event A: the selected animal is dogs
Event B: the selected animal lives inside

CHECK: $P(A|B) = P(A)$ and $P(B|A) = P(B)$
If both equal, they are independent.

$P(A|B) = P(A)$

$0.47 \approx \frac{7}{15} \neq \frac{24}{40} = 0.6$

$P(B|A) = P(B)$

$0.29 \approx \frac{7}{24} \neq \frac{15}{40} = 0.375$

∴ No, not independent

21. What is the probability that a randomly chosen child likes chocolate?

This says... what is prob. of $P(\text{chocolate} | \text{child}) = 8/29$

22. What is the probability that a randomly chosen person likes strawberry?

27/82

What flavor of ice cream do you want?				
	Children	Teens	Adults	Total
Chocolate	8	12	10	30
Vanilla	6	9	10	25
Strawberry	15	6	6	27
Total	29	27	26	82

23. What is P(is a teen | vanilla)?

9/25

24. What is the probability you are a child given you like strawberry?

$15/27 = 5/9$

25. Are the events A and B indepdendent?

Event A: the selected person is a child
Event B: the selected person likes vanilla

$P(A|B) = P(A)$

$0.24 = \frac{6}{25} \neq \frac{29}{82} \approx 0.35$

$P(B|A) = P(B)$

$0.21 \approx \frac{6}{29} \neq \frac{25}{82} \approx 0.30$

∴ NO, again

You are only ready for the test when you have completed all questions correctly, could explain them to another person, and can begin and finish each without assistance!!! STUDY!